Sectoral Heterogeneity and Monetary Policy*

Jonathan Kreamer†

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Abstract

Since sectors differ in their sensitivity to interest rates, monetary policy produces inefficient sectoral fluctuations. In a model with sectoral heterogeneity, I show that policymakers should weight sectors proportionally to their interest elasticities, account for dynamic demand effects from durable goods, and systematically utilize forward guidance to reduce sectoral volatility. A calibrated model confirms these recommendations, and finds that neglecting sectoral volatility produces substantial welfare losses. The optimal policy is well-approximated by a policy that stabilizes a sectorally-weighted measure of inflation, plus lags of past durable inflation. I calculate sectoral labor wedges to assess historical U.S. monetary policy.

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†Florida State University, Department of Economics; email: jkreamer@gmail.com

1 Introduction

Economists have long understood that sectors of the economy differ in their sensitivity to interest rates, with durable consumer goods and investment being especially sensitive to interest rates.[1] Given this differential sensitivity, the conduct of monetary policy implies fluctuations in sectoral production which may be quite inefficient. Nevertheless, policy-makers generally focus on aggregate factors when setting monetary policy, considering these sectoral dislocations second-order. In this paper, I argue that this is a mistake, and that monetary policy should take sectoral heterogeneity into account.

I develop the argument using both a simplified analytical model with multiple goods, and a calibrated quantitative model with two sectors. I allow sectors to differ in the durability of produced goods, which produces differential interest sensitivity of demand across sectors. I find four main results. First, in the presence of sectoral heterogeneity and short-run imperfect substitution between sectors, monetary policy alone generically fails to achieve the first best. Second, it is optimal to stabilize a weighted average of sectoral labor wedges, where weights are proportional to sectoral interest elasticities. Third, policy should account for dynamic demand effects from durables, as greater durable production increases the stock of durables, decreasing future demand for new durables. Fourth, if central banks can commit to a future path of policy, they should systematically use forward guidance, since future interest rates produce smaller sectoral fluctuations than current interest rates. I prove these results in an analytical model, and then show that they hold qualitatively (and to a quantitatively significant degree) in a calibrated quantitative model. I also show that the optimal policy rule following a simple demand shock is well-approximated by targeting a measure of inflation that places equal weights on durable goods, nondurable goods, and a weighted average of past durable inflation.[2]

This paper consists of three parts. In section 2, I work with an analytical model with many goods and a general utility function. Goods differ in their interest sensitivity both because of differential durability and differential interest-elasticity of substitution from the utility function. For tractability I assume a particular restrictive form of nominal rigidity: that prices are fixed for N periods and flexible thereafter. This allows the derivation of simple expressions for optimal monetary policy that yield intuitive interpretations. I find that optimal policy can be stated in terms of sectoral output shares, labor wedges, and interest elasticities. The baseline analysis proceeds under further simplifying assumptions, but I show in the appendix that many of these can be relaxed without qualitatively

[1] I review the literature on sectoral interest elasticities in section 4.2.

[2] This is assuming a durable GDP share of $1/8$, so equal weights between the sectors implies weighing current durable inflation seven times more than implied by GDP share.

changing the results.

The first case I consider is that prices are fixed for one period only. I find that optimal policy satisfies what I call the static optimal policy rule:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

where γ^j is the GDP share of sector j, ϵ_R^j is the interest elasticity of demand, and τ^j is the labor wedge. If $\tau^j = 0$ for all sectors is feasible, then this is the solution; however, this will not generally be possible, since fixed prices implies fixed relative prices between sectors, whereas most shocks imply relative price movements. When stabilization of every sector is infeasible, the central bank must trade off stabilizing some sectors against others when setting interest rates. One might naively think that it is optimal to weight sectors by their GDP shares alone, as is done under inflation targeting.[3] However, this expression implies that sectors should be weighted by the interest elasticity, implying that more interest-sensitive sectors should receive a higher weight in setting monetary policy.

I next consider cases with $N > 1$, so that the cental bank can set interest rates for multiple periods. Now the results depend on whether the central bank can commit to future policy. Without commitment, I derive an optimal policy rule that balances current labor wedges weighted by interest elasticities, and future labor wedges weighted by current interest elasticities and sectoral durability. This captures that production of durable goods in the current period increases the stock of durables in future periods, thereby reducing future demand in those sectors. I call this the durable overhang effect.[4]

I also consider cases with commitment. Commitment allows the central bank to set the future path of interest rates for the next N periods, rather than just the current rate. This allows the central bank to utilize forward guidance, i.e. to commit to changing future interest rates to affect demand today. If future interest rates had equivalent effects as current rates, then this would make no difference. More precisely, I calculate an optimal policy rule that is as in the case without commitment plus a term that captures the covariance of sectoral interest elasticities and the difference between interest elasticity with respect to current and future interest rates. In other words, if relative sectoral demand responds equivalently to current and future interest rates, there is no benefit from commitment. However, if relative sectoral demand responds differentially to future and

[3]There is an analogy between labor wedges and inflation, since the labor wedge is proportional to the desired markup, and to the output gap in a linearized one-sector model.

[4]Two recent papers, Rognlie et al. (2014) and Beaudry et al. (2014), discuss an analogous effect, although their concern is with aggregate demand rather than sectoral factors.

current interest rates, there are gains from using forward guidance. I argue that under standard theory current interest rates produce greater sectoral volatility, since durable goods are more sensitive to current interest rates than to future interest rates, whereas nondurable goods show no such difference. This implies that if a central bank is able to commit to a policy path, it should systematically use forward guidance in the conduct of monetary policy.

In section 3, I replace the N-period fixed price assumption with Calvo pricing. This allows a more realistic treatment of inflation and the calibration of a quantitative example. I calibrate a two-sector model, with one sector correponding to durable goods (e.g. consumer durables and residential investment) and the other sector corresponding to nondurable goods. I confirm that durable goods are much more interest-sensitive than nondurable goods in the model, and that durable goods are more sensitive to current than to future interest rates.

I then compare the performance of various monetary policy rules following a simple demand shock — a shock that in a one-sector model could be fully offset by lowering the interest rate. Since the model has two sectors that differ in interest sensitivity, under flexible prices this shock will cause relative appreciation of the durable goods in response to the lower interest rate. With nominal rigidities, the relative price cannot adjust sufficiently to achieve this result, and thus there is an inefficient shift in relative sectoral demand. If the central bank seeks to stabilize aggregate output, the result will be a relative expansion of the durable sector, and a contraction of the nondurable sector.

I compare several classes of policy rules. Among rules that make reference to current aggregate variables only, I find that stabilizing labor performs better than stabilizing inflation, and that the optimal rule in aggregates places about equal weight on inflation and labor. Wages turn out not to matter for setting optimal policy. When the policy rule can place different weights on inflation in each sector, it is optimal to weight the durable sector a bit more than the nondurable sector (or about 7 times more than under inflation targeting). There is little added benefit in targeting labor or wages when the central bank can target sectoral inflation, suggesting sectoral inflation is close to optimal for policies that set interest rates based on current variables only.

Finally, I characterize the optimal path of interest rates following the shock, corresponding to the full commitment optimal policy. As suggested by the analytical model, I find that it is optimal to shift the path of interest rates forward in time, making use of forward guidance. This rule outperforms optimal sectoral inflation targeting to a small but significant degree. I search for a simple policy rule that approximates these dynamics by looking at policy rules in current and lagged sectoral inflation. I find that the optimal path can be closely approximated by a policy rule that places equal weight on durable and nondurable inflation in the current period, and then $(1/2)^k$ as much weight on the kth lag of durable inflation. That is:

$$\frac{1}{2}\pi_t^d + \frac{1}{2}\pi_t^c + \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k+1}\pi_{t-k}^d = 0$$

This implies that if the economy has recently been experiencing above-target durable inflation, it is optimal to lower the current inflation target, and vice versa.

Lastly, in section 4 I apply the policy rules derived in section 2 to assess the historical conduct of U.S. monetary policy. To do so, I construct historical measures of sectoral labor wedges, and survey the literature on sectoral interest elasticities. I then compute the elasticity-weighted labor wedge, and compare it to an aggregate labor wedge that neglects differential sectoral sensitivity. I find that the aggregate labor wedge has lower historical volatility than any of the sectoral wedges, supporting the view that the Fed has historically focused on aggregate factors. Comparison to the elasticity-weighted optimal static policy rule suggests that the Fed should be more aggressive in using monetary policy to reduce fluctuations, since the more interest sensitive sectors also more procyclical. Perhaps surprisingly, the results do not support the view that monetary policy was too loose during the 2002 – 2006 period. This is because, although there was high relative inflation in the construction sector during this period, the labor wedge was not particularly low relative to the economy at large.

Literature. This paper is most closely related to the literature on optimal monetary in models with durable goods and substantial price rigidity across sectors. Erceg and Levin (2006) analyze a two-sector Calvo model with durable goods that is similar to the one I consider in section 3. They show that their model fits a VAR quite well. They compare various simple policy rules, and find that inflation targeting performs poorly, while a policy that targets the aggregate output gap and a wage-price rule each perform well. In a recent working paper, Barsky et al. (2016) study monetary policy in a two-sector model with durable goods. They find that durable goods are highly interest-sensitive, and that the output gap is heavily dependent on durable inflation. They argue that monetary policy should place a heavy weight on durable inflation to stabilize the output gap, consistent with my findings. Relative to these papers, I focus on deriving optimal policy in a simpler model with a wider set of goods. This allows me to highlight the role of sectoral interest elasticity, and derive results concerning durable overhang, the role of commitment, and forward guidance that others have not found due to focusing only on comparing numerical methods and simple rules.

Beyond durable goods, there are a number of papers that consider optimal monetary policy with sectoral heterogeneity. Mankiw and Reis (2003) ask what price index a central bank should target. This question is quite similar to that considered in this paper, but the dimensions of sectoral heterogeneity considered are quite different. They allow sectors to differ by size, cyclical sensitivity, size sectoral shocks, and price flexibility, but not by interest elasticity (except implicitly through price flexibility). Moreover, they focus on the limiting case of infinite risk aversion, under which sectoral volatility vanishes in their welfare function, whereas I am chiefly concerned with this volatility.

In a similar vein, several papers have examined sectoral heterogeneity in the degree of price stickiness. Aoki (2001) study a two-sector model, with one flexible and one sticky sector, and argues that it is optimal to target inflation in the sticky price sector only. Carvalho et al. (2006) analyze sectoral heterogeneity in price stickiness from a positive perspective, and argue that it increases sluggishness of overall inflation. Bouakez et al. (2009) estimate a multisector DSGE model, and conclude that there is substantial sectoral heterogeneity in price stickiness.

Continuing the theme of heterogeneity in price stickiness, Barsky et al. (2007) document that a two-sector model with flexible durable prices implies negative sectoral co-movement following a monetary policy shock. Since this is at odds with the data, they term this the co-movement puzzle. Several papers have tried to resolve this puzzle by various methods, while others have simply assumed that the prices of durable goods are sticky as well. Notably, Bouakez et al. (2011) claim that including intersectoral demand linkages eliminates the comovement puzzle.

This paper is also related to the literature on monetary unions. The problem in a monetary union is that a single monetary policy is applied to heterogeneous regions, which is analogous to the problem of a single monetary policy with heterogeneous sectors. Benigno (2004) analyzes optimal monetary policy in a currency union, and finds that an inflation target that weights countries proportionally to their price rigidity is close to optimal.

Recently Bils et al. (2013) exploit the greater cyclical sensitivity of durable goods to test the "Keynesian labor demand hypothesis", i.e. that producers respond to low demand by cutting production rather than prices. They confirm that durable goods are highly cyclically sensitive in the data, and find countercyclical markups consistent with significant price stickiness in durable sectors. They conclude that this supports the Keynesian labor demand hypothesis.

2 Model without Inflation

I start by deriving the main results in a simple model. The baseline model abstracts from inflation and uncertainty, which allows the derivation of simple and interpretable expressions for optimal policy. I introduce inflation in the quantitative model in section 3, and uncertainty in appendix B.

2.1 Flexible Prices

The model is set in discrete time ($t = 0, 1, \dots$). There are N_j consumption goods, indexed by $j \in 1, \dots, N_j$. There is a unit measure of households, and a unit measure of firms in each sector. There is no capital, and the only asset is a nominal bond which is in zero net supply.[5]

Households. Households have lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \theta_t [u(\vec{c}_t) - v(n_t)] \tag{1}$$

where θ_t is a demand shock, $\vec{c}_t$ is a vector of consumption goods, and n_t is labor.[6] I assume that $u(\cdot)$ is strictly increasing and concave, and that $v(\cdot)$ is strictly increasing and convex.

Consumption goods may be durable. In particular, I assume that good j depreciates at the rate $\delta^j \in \{0, 1\}$. Goods in sector j have price p_t^j. Households supply labor n_t in return for wages w_t. They own firms and receive profits π_t^j from firms in sector j. There is a nominal bond that yields gross return R_{t+1} in period $t+1$. These assumptions yield household budget constraint:

$$a_{t+1} + \sum_j p_t^j \left[c_t^j - (1 - \delta^j) c_{t-1}^j \right] = w_t n_t + \sum_j \pi_t^j + R_t a_t \tag{2}$$

Households maximize utility (1) subject to budget constraint (2). Household optimal-

[5]Since there is a representative household, the only role of this bond is to define the interest rate of the economy, which is the mechanism through which monetary policy is conducted.

[6]The model can be generalized to different types of labor.

ity conditions are:

$$w_t = \frac{v_{n_t}}{\lambda_t} \tag{3}$$

$$\lambda_t = \frac{\beta\theta_{t+1}}{\theta_t} R_{t+1}\lambda_{t+1} \tag{4}$$

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \left(\frac{1-\delta^j}{R_{t+1}}\right) p_{t+1}^j \tag{5}$$

where (3) and (4) are standard labor supply and Euler equations, and (5) is an asset-pricing condition for potentially durable consumption goods.

Interest-elasticity of demand. A central feature of the analysis is the differential sensitivity of sectoral demand to interest rates. Thus it is important to note that interest sensitivity is increasing in a sector's durability. To see this, suppose the utility function is separable. Then log-linearizing (4) and (5) and combining them yields:

$$\sigma^j \hat{c}_t^j = -\hat{\lambda}_{t+1} + \hat{\theta}_t - \hat{\theta}_{t+1} - \hat{p}_t^j - \hat{R}_{t+1} + \left(\frac{1-\delta^j}{r+\delta^j}\right)\left(\pi_{t+1}^j - \hat{R}_{t+1}\right)$$

where $r = R - 1$ is the steady state net interest rate.

Deviations in the interest rate $\hat{R}_{t+1}$ enter this expression in two places. The first is the standard aggregate demand effects operating through intertemporal substitution, as expressed in the Euler equation. This applies to all goods in proportion to their IES, regardless of durability. The second appearance captures additional demand effects on durable goods, which occur because durable goods are a means of saving as well as providing current consumption. A reduction in the interest rate R_{t+1} raises the present value of future income, which raises the value of durable good purchases to the household, and causes the household to demand more durable goods.

To focus on the effects of interest rates, suppose that demand shocks, changes in prices, and future income shocks are all zero. Then we have a partial equilibrium analog of the interest elasticity of demand for good j of:

$$-\frac{\hat{c}_t^j}{\hat{R}_{t+1}} = \left(\frac{R}{r+\delta^j}\right)\frac{1}{\sigma^j}$$

To see how much difference this can make to the interest elasticity, suppose that a good has a depreciation rate of 10% and that the interest rate is 5%, as in the calibration in Erceg and Levin (2006). Then the interest elasticity of demand for that good would be about 6 times greater than a non-durable good with the same IES.[7]

Firms. There is a unit interval of firms in each sector that produce using technology:

$$y_t^j = f_t^j(n_t^j) \tag{6}$$

where f_t^j satisfies the Inada conditions. Labor is perfectly substitutable across sectors. Thus firm profits are:

$$\pi_t^j = p_t^j y_t^j - w_t n_t^j \tag{7}$$

which implies optimal labor demand:

$$p_t^j = \frac{w_t}{f_{n_t}^j} \tag{8}$$

Equilibrium. The market clearing conditions are:

$$a_t = 0 \tag{9}$$

$$c_t^j = y_t^j + (1 - \delta^j) c_{t-1}^j \tag{10}$$

$$n_t = \sum_j n_t^j \tag{11}$$

Equilibrium is defined as follows:

Definition 1 (Flexible price equilibrium)**.** Given initial goods and demand shock $\{\vec{c}_{t-1}^j, \theta_t\}$, the equilibrium is a path of prices $\{p_t^j, w_t, R_{t+1}\}$ and quantities $\{c_t^j, n_t^j, n_t, y_t^j, \pi_t^j, a_t)\}$ that satisfy household conditions (3) - (5), firm conditions (6) - (8), and market clearing conditions (9) - (11).

The definition above does not pin down the level of nominal prices, since all equilibrium expressions will continue to hold if p_t^j and w_t are multiplied by a constant factor. To specify equilibrium, we choose good 1 to be the numeraire. Further, we will henceforth assume that good 1 is non-durable, so that $\delta^1 = 1$. This is a convenient assumption for expository purposes, but does not affect any results. Note that this implies that R is now the *real* interest rate in terms of good 1.

[7] This calculation neglects changes in relative prices that would occur in general equilibrium, and thus will be greater than the true elasticity of demand. On the other hand, assuming $\delta^j = 0.1$, a 1% change in demand for durables would imply a 10% change in the demand for *new* durables.

2.2 One-period fixed prices

To analyze the consequences of differential sectoral interest elasticity, we need some sort of sticky prices so that monetary policy has real effects. The simplest form of nominal rigidity is that the nominal prices of goods in all sectors are fixed for the current period, and flexible thereafter. Thus $p_t^j = \bar{p}^j$ for $t = 0$.

Following the normal convention in New Keynesian models, I assume that output is wholly demand determined. Thus all household optimality conditions continue to hold, but firm labor demand conditions (8) may not be satisfied:

$$w_t \neq p_t^j f_{n_t}^j$$

Since it is only the relative prices that matter for equilibrium, fixed prices in N sectors for one period determines $N-1$ real variables. Since we have discarded N equilibrium conditions, we need one more expression to specify equilibrium. Thus I assume that the central bank chooses the real interest rate R_{t+1}.[8]

In periods $t \geq 1$, equilibrium is defined according to the flexible price equilibrium defined in section 2.1

Statement of Optimal Policy Problem. We next formulate the optimal policy problem. Consider the problem of the planner in period t, given fixed prices $\bar{p}_t^j$ and previous period goods $\vec{c}_{t-1}$. The next period value function $V_{t+1}(\vec{c}_t, \theta_{t+1})$ is known. The social planner then chooses R_{t+1} to maximize the welfare of the representative household.

First we define how demand for good j depends on interest rates. Since starting the next period the economy will be in the flexible price equilibrium, the functions $\vec{c}_{t+1}(\vec{c}_t, \theta_{t+1})$ and $\vec{p}_{t+1}(\vec{c}_t, \theta_{t+1})$ are known. Then we can combine (5) with fixed prices and the Euler equation to obtain:

$$u_{c_t^j} = \bar{p}_t^j u_{c_t^1} \left(1 - \left(\frac{1-\delta^j}{R_{t+1}}\right) \frac{p_{t+1}^j}{\bar{p}_t^j}\right) \tag{12}$$

where we make use of the fact that $\lambda_t = u_{c_t^1}$. This gives us N equations in N unknowns, with the equation for good 1 simply being the standard consumption Euler equation. Thus this expression defines $\vec{c}_t(R_{t+1})$.

Now we turn to expressing welfare as a function of $\vec{c}_t$. First we define aggregate labor

[8] I assume that the interest rate is chosen such that demand for new durable goods is always non-negative.

supply as a function of $\{\vec{c}_t\}$ by inverting the production function in each sector:

$$n_t\left(\vec{c}_t\right) = \sum_j \left(f_t^j\right)^{-1} \left(c_t^j - \left(1-\delta^j\right) c_{t-1}^j\right) \tag{13}$$

Now we can express the optimal policy problem as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right) \right\}$$

where $V_{t+1}(\vec{c}_t)$ is the next period value function.[9]

This problem is well-defined since we are considering a case with flexible prices starting in the next period. Thus $V_{t+1}(\vec{c}_t)$ is just the lifetime utility of the representative household that enters the flexible price equilibrium with initial goods $(\vec{c}_t)$.

Optimal Policy. The following proposition describes the optimal monetary policy:

Proposition 1 (Static Optimal Policy Rule)**.** *The optimal choice of R_{t+1} satisfies:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

where $\epsilon_R^j = -\frac{dy_t^j}{dR_{t+1}} \frac{R_{t+1}}{y_t^j}$ is the interest-elasticity of demand for new production in sector j, $\gamma_t^j = \frac{p_t^j y_t^j}{\sum_j p_t^j y_t^j}$ is the GDP share of sector j, and $\tau_t^j = 1 - \frac{w_t}{p_t^j f_{n_{t+1}}^j}$ is the labor wedge in sector j.

Proof. See appendix D. □

That is, the optimal policy is to set the interest rate so that the weighted average labor wedge is zero, where sectoral labor wedges are weighted by the size of the sector and the interest-elasticity of that sector. I call this the *static policy rule*, since it reflects the static tradeoffs associated with setting interest rates.

The sectoral labor wedges captures the degree to which there is excess or deficient demand in a given sector. $\tau_t^j > 0$ corresponds to deficient demand, because it indicates that output is below the optimal level. τ_t^j can also be interpreted as the percent deviation from the optimal markup (here zero) in sector j. Thus $\tau_t^j > 0$ indicates an excess markup, suggesting that firms in sector j would like to lower prices, but are prevented from doing

[9]This problem formally includes non-negativity constraints on the production of new goods. We do not need to consider these constraints explicitly because the unconstrained solutions will necessarily be interior since the production functions satisfy the Inada conditions.

so by the nominal rigidity. This is consistent with the interpretation of deficient demand in the sector.

If we added a marginal amount of price flexibility, we would find that sectoral inflation was inversely proportional to the sectoral labor wedge, i.e. $\pi_t^j \propto -\tau_t^j$.[10] Thus an alternative interpretation of the policy rule is that it is optimal to target inflation, where the inflation target weights prices by the sectoral interest elasticity. Given that empirical estimates suggest substantially higher interest elasticity for durable goods and particularly housing, this suggests policymakers should place greater weight on inflation in these sectors when setting policy.[11]

To understand the derivation of the static policy rule, simply note that $\gamma_t^j \tau_t^j$ is proportional to the marginal benefit of increasing demand for good j. Optimal monetary policy consists of setting the interest rate such that the net marginal benefit of cutting the rate equals the marginal cost of doing so. The marginal benefit is the marginal value of increasing demand in every sector that has $\tau_t^j > 0$, i.e. that is experiencing insufficient demand, times the sensitivity of demand to a marginal change in interest rates. The marginal cost is defined likewise for those sectors with $\tau_t^j < 0$.

Welfare Properties of Optimal Policy. Now we discuss the welfare properties of optimal policy. The results are summarized in the following proposition:

Proposition 2 (Welfare Properties of Equilibrium under Optimal Policy)**.** *Suppose the economy is initially at a Pareto Optimum and experiences a shock. The resulting equilibrium under optimal policy is Pareto Optimal only when $\bar{p}^j = p_t^{j,flex}$ for all j. A sufficient condition for this is linear production in each sector together with no idiosyncratic sectoral supply shocks.*

Proof. See appendix D. □

Proposition 2 implies that monetary policy alone is able to achieve the first best only in very particular circumstances. Since most economic shocks will imply some changes in relative prices in the flexible price equilibrium, it follows that the constraint of fixed relative prices will bind. The exception is the case of linear production in each sector, which implies that relative prices do not change in response to shocks, with the exception of sectoral productivity shocks. This is an extreme assumption, since it implies perfect substitution of productive factors between sectors.

[10] For example, we could suppose that a fraction ϕ of firms in each sector are allowed to reset their prices that period, and take the limit as $\phi \to 0$. Since prices are flexible the next period, the optimal pricing rule is to set $p_t^j = w_t / f_{n_t}^j = \left(\frac{1}{1-\tau_t^j}\right) p_t^j$, assuming an optimal markup of zero.

[11] I review estimates of sectoral interest elasticities in section 4.2.

Now that we have established the second-best nature of pure monetary stabilization, we turn to characterizing optimal monetary policy.

2.2.1 Tradeoff between aggregate and sectoral stabilization.

To characterize optimal policy, it is useful to express the policy rule in the following form:

Proposition 3 (Static Optimal Aggregate Sectoral Tradeoff)**.** *The static policy rule can be written as:*

$$\tau^y + \left(\epsilon_R^y\right)^{-1} \sum_j \gamma^j \left(\epsilon_R^j - \epsilon_R^y\right) \left(\tau^j - \tau^y\right) = 0$$

where:

$$\epsilon_R^y = \sum_j \gamma^j \epsilon_R^j$$
$$\tau^y = \sum_j \gamma^j \tau^j$$

Proof. See appendix D. □

This representation of the static optimal policy rule highlights the tradeoff between aggregate and sectoral stabilization. τ_t^y is the aggregate labor wedge, and ϵ_R^y is the aggregate interest-elasticity of demand. Since γ^j are sectoral weights, the second term is the covariance between sectoral interest-elasticity and labor wedges.

This expression indicates that when relatively interest-sensitive sectors are experiencing relatively high demand, it is optimal to tolerate deficient aggregate demand, rather than excessively stimulate these sectors by cutting interest rates further. It further indicates that aggregate stabilization is optimal when either all sectors are equally interest-sensitive, or all sectors have zero labor wedges when the aggregate labor wedge is zero, or when there is no correlation between interest elasticity and relative labor wedges. Since none of these cases is likely to hold in practice, we can say that the general result is that it is rarely optimal to target a zero aggregate labor wedge while neglecting sectoral considerations.

2.3 N-period Fixed Prices without Commitment

Now we extend the model to the case where prices may be sticky for multiple periods. Suppose that prices of goods in all sectors are fixed for N periods, and flexible thereafter. Thus $p_t^j = \bar{p}^j$ for $t < N$. As in the previous section, output is demand determined, household optimality conditions hold, and firm labor demand conditions may fail to hold for $t \leq N$. The central bank now chooses interest rates in every period with fixed prices.

Since the central bank is now choosing interest rates for several periods, we have to consider whether the central bank is able to commit to a future path of interest rates. I assume in this section that the central bank cannot so commit, and is limited to only choosing the current interest rate. I consider commitment in the next section.

Statement of Optimal Policy Problem. We formulate the optimal policy problem analogously to the previous section. Consider the problem of the planner in period t, given fixed prices $\bar{p}_t^j$ and previous period goods $\vec{c}_{t-1}$. The next period value function $V_{t+1}(\vec{c}_t, \theta_{t+1})$ is known. The social planner then chooses R_{t+1} to maximize the welfare of the representative household. Equations (12) and (13) continue to hold.

Now we can express the optimal policy problem without commitment recursively as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right) \right\}$$

where $V_{t+1}(\vec{c}_t)$ is the next period value function.

For $t = N$, this problem is just the same as the one-period fixed price problem analyzed in the previous section. For $t < N$, the problem is different, and the next period value function V_{t+1} is defined recursively. This problem is well-defined since V_N is well-defined, and we can then define each previous period's problem by backward induction, where $V_{t+1}(\vec{c}_t)$ represents the value function under future optimal policy. Since the social planner lacks commitment, it cannot credibly promise to alter future policy, and thus takes $V_{t+1}(\vec{c}_t)$ as given in its choice of R_{t+1}.

Optimal Policy. The following proposition describes the optimal monetary policy:

Proposition 4 (Optimal Policy Without Commitment)**.** *The optimal choice of R_{t+1} satisfies:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}}\right) \tau_{t+1}^j$$

where $\epsilon_R^j = -\frac{dy_t^j}{dR_{t+1}} \frac{R_{t+1}}{y_t^j}$ is the interest-elasticity of demand for new production in sector j, $\gamma_t^j = \frac{p_t^j y_t^j}{\sum_j p_t^j y_t^j}$ is the GDP share of sector j, and $\tau_t^j = 1 - \frac{w_t}{p_t^j f_{n_{t+1}}^j}$ is the labor wedge in sector j.

Proof. See appendix D. □

The left-hand side of the optimal policy expression is the marginal value of cutting interest rates due to the effects on demand in the current period. When the right-hand side is zero, this is just the same as the static policy rule that prevails under one-period fixed prices.

The right-hand side is the dynamic marginal cost of cutting interest rates, deriving from entering the next period with a higher stock of durable goods. Holding desired consumption fixed, a higher initial stock of durable goods implies lower demand for new output. If there is overall deficient demand in durable sectors in the next period, as indicated by $\tau^j_{t+1} > 0$, then cutting interest rates in the current period carries a cost since it reduces effective future demand in these sectors. Thus it will be optimal to not cut interest rates as much as static considerations alone would imply, allowing a current weighted output gap $\sum_j \epsilon^j_R \gamma^j_t \tau^j_t > 0$. I call this the *durable overhang effect*.

2.4 N-period fixed prices with commitment

The analysis of optimal policy in section 2.3 assumed no commitment by the central bank. This turns out to matter because demand responds differentially to current and future interest rates. Differential sectoral sensitivity is greater with respect to current interest rates than with respect to future interest rates. Thus to minimize sectoral distortions, it is optimal to use forward guidance to minimize fluctuations in current interest rates.

First we formulate the problem. The economy enters period 0 with previous consumption vector $\vec{c}_{t-1}$. The monetary authority then chooses interest rates for the next N periods $\{R_s\}_{s=1}^N$. We may state the problem as:

$$V_0\left(\vec{c}_{-1}\right) = \max_{\{R_s\}_{s=1}^N} \left\{ U_0\left(\vec{c}_1, \vec{c}_0\right) + \sum_{t=1}^{N-1} \frac{\beta^s \theta_t}{\theta_0} U_t\left(\vec{c}_t, \vec{c}_{t-1}\right) + \frac{\beta^N \theta_N}{\theta_0} V_N\left(\vec{c}_{N-1}\right) \right\}$$

where $U_t\left(\vec{c}_t, \vec{c}_{t-1}\right) = u\left(\vec{c}_t\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right)$ is the period utility function, and V_N is the flexible price value function. $\vec{c}_t$ are period demand functions.

Optimal Policy. We now turn to solving the optimal policy problem. First I show that demand in period t depends only on future interest rates, not on past interest rates.

Lemma 1 (Dependence on future interest rates only)**.** *Period t demand is a function only of fixed relative prices and the future path of interest rates, $\vec{c}_t\left(\vec{p}, \{R_s\}_{s \geq t+1}\right)$. In particular, c^j_t does not depend on past interest rates, so $\epsilon^{y^j_t}_{R_k} = 0$ for $k \leq t$.*

Proof. See appendix D. □

This lemma might be surprising, as one might think that cutting current interest rates affects future demand through changing the stock of future durables. That is, a lower interest rate in period t implies a greater stock of durables in period $t+1$. But with fixed prices and a given interest rate, there is simply no mechanism through which this greater stock of durables increases demand. It simply reduces effective demand for new durables, with the loss in income to households from lower production exactly offsetting the higher income from a greater stock of durables.

With Lemma 1 in hand, we use similar techniques to the last two sections to derive an expression for optimal policy:

Proposition 5 (Optimal policy with commitment)**.** *Optimal policy satisfies:*

$$\sum_{t=0}^{k-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_k}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \tau_{t+1}^j \left(\frac{1-\delta^j}{R_{t+1}} \right)$$

is the durable overhang-augmented sectoral labor wedge.

Proof. See appendix D. □

Simplification of Optimal Policy Expression. The expression derived in Proposition 5 is difficult to interpret by itself. Start with the choice of R_1, i.e. the interest rate in the initial period. The expression is just the same as the without commitment:

$$\sum_j \gamma_0^j \epsilon_{R_1}^{y_0^j} \chi_0^j = 0$$

Intuitively, since the choice of R_1 only affects demand in period 0, the presence or absence of commitment does not matter.[12]

What about for the choice of R_k with $k > 1$? Here we can simplify by taking the difference between the optimality expressions for R_k and R_{k-1}. The key is that the elasticities of demand with respect to interest rates s periods ahead are the same for all $s > 1$, as proved in the following Lemma.

Lemma 2 (Symmetric effects of forward guidance)**.** $\epsilon_{R_k}^{y_t^j} = \epsilon_{R_\ell}^{y_t^j}$ *for* $k, \ell \in [t+2, N]$.

[12] Of course, this does not imply that the choice of R_1 will be the *same* as it would be without commitment. Since the choice of future interest rates will differ, current and future labor wedges will differ as well. It is merely the optimal policy expression that is unchanged.

Proof. See appendix D. □

With Lemma 2 in hand, we can derive a simplified form of the optimality condition.

Proposition 6 (Simplified Optimal Policy Expression)**.** *For $k \in [2, N]$, combining the optimal choice of $\{R_k, R_{k-1}\}$ yields optimality condition:*

$$\sum_j \gamma^j_{k-1} \epsilon^{y^j_{k-1}}_{R_k} \chi^j_{k-1} = \frac{R_{k-1} y_{k-2}}{y_{k-1}} \sum_j \gamma^j_{k-2} \left(\epsilon^{y^j_{k-2}}_{R_{k-1}} - \epsilon^{y^j_{k-2}}_{R_k} \right) \chi^j_{k-2} \tag{14}$$

Proof. See appendix D. □

The key term here is $\epsilon^{y^j_{k-2}}_{R_{k-1}} - \epsilon^{y^j_{k-2}}_{R_k}$. For example, a positive term indicates that sector j is more sensitive to current interest rates than to future interest rates. Thus the righthand side of (14) is the covariance between the sectoral dynamic labor wedge and the sectoral relative sensitivity to current interest rates. When this term is 0, the optimality expression is just as in the no-commitment case. Thus the deviation from the no-commitment benchmark is proportional to the covariance between dynamic sectoral output gaps and differential sectoral interest elasticities with respect to current and future interest rates.

This expression governs the optimal use of forward guidance. The lefthand side is negative when forward guidance is employed, because this indicates that R_k is set lower than implied by the no-commitment policy rule in period $k-1$. This occurs when the righthand side is also negative — i.e., when past labor wedges are negatively correlated with relative sensitivity to current interest rates.

Sectoral variation in sensitivity to current and future interest rate. The optimal policy expression above suggests that it is optimal to routinely use forward guidance in the conduct of monetary policy as long as there is some systematic relationship between sectoral output gaps and sectoral relative sensitivity to current and future interest rates. In fact this is the case — generally speaking, durable goods are less sensitive to future interest rates than they are to current interest rates, whereas nondurable goods are equally sensitive to current and future interest rates.

The intuition for this is straightforward. Consider the pricing equation for good j:

$$u_{c^j_t} = \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j u_{c^1_t}$$

The current interest rate R_{t+1} appears directly in the expression, with its influence increasing in durability. By contrast, future interest rates enter through the Euler equation, i.e. through $u_{c_t^1}$. That is, cutting future interest rates lowers $u_{c_{t+1}^1}$, which in turn lowers $u_{c_t^1}$. This channel works independently of durability. We can prove this in the case of separable utility:

Proposition 7 (Durability and Forward Guidance)**.** *Suppose that $u(\vec{c}_t)$ is separable in c_t^j. Then we have:*

$$\epsilon_{R_{t+1}}^{y_t^j} - \epsilon_{R_{t+2}}^{y_t^j} = \left(\frac{1-\delta^j}{R_{t+1}}\right) \epsilon_{R_{t+1}}^{y_t^j}$$

and optimal policy satisfies:

$$\sum_j \gamma_{k-1}^j \epsilon_{R_k}^{y_{k-1}^j} \chi_{k-1}^j = \frac{R_{k-1} y_{k-2}}{y_{k-1}} \left[\sum_j \gamma_{k-2}^j \left(\frac{1-\delta^j}{R_{k-1}}\right) \epsilon_{R_{k-1}}^{y_{k-2}^j} \chi_{k-2}^j\right]$$

Proof. See appendix D. □

Proposition 7 suggests that the durable goods are relatively more sensitive to *current* interest rates than to *future* interest rates. This suggests that when durable goods are relatively overstimulated, the central bank should promise to cut future interest rates (instead of current rates), and when durable goods suffer relatively low demand, the central bank should promise to raise future interest rates and cut current interest rates further instead.

3 Quantitative Model with Inflation

The previous section derived general results for optimal policy under a particularly restrictive price setting assumption: that prices were fixed for several periods, and then flexible thereafter. This assumption enabled the derivation of simple and clear expressions for optimal policy with straightforward interpretations. However, abstracting from inflation has two shortcomings. First, it is unclear whether the general qualitative results derived in the previous section are robust to the inclusion of inflation, and particularly costs of inflation. Second, it is difficult to assess the quantitative importance of sectoral considerations in such a simplified model. To address these concerns, this section presents a calibrated two-sector Calvo model, and verifies the quantitative significance of the results from the preceding section.

3.1 Model

For convenience and consistency with the model presented in section 2, I stick with the assumption of perfect foresight. This allows the model to be solved exactly. There are two goods: a nondurable consumption good c, and a durable consumption good d. I denote the relative price between the sectors by $q_t = p_t^d / p_t^c$, and give other prices in terms of the non-durable good. Household utility has functional form:

$$\sum_{t=0}^{\infty} \beta^t \theta_t \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{d_t^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^{1+\zeta}}{1+\zeta} \right)$$

Note that utility is separable in both consumption goods, constant elasticity is assumed throughout, and both consumption goods have the same intertemporal elasticity of substitution. Under these assumption household optimality conditions are:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \frac{\beta \theta_{t+1}}{\theta_t} R_{t+1} \tag{15}$$

$$q_t = \chi \left(\frac{c_t}{d_t} \right)^{\sigma} + \frac{1-\delta}{R_{t+1}} q_{t+1} \tag{16}$$

$$\psi n_t^{\zeta} = w_t c_t^{-\sigma} \tag{17}$$

Supply Side. Suppose that in sector j there is a unit interval of intermediate good firms indexed by i.[13] The output of intermediate good firms is combined by a Dixit-Stiglitz aggregator firm to produce final goods:

$$y_t^j = \left(\int_i \left(y_{it}^j \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Intermediate goods are produced by firms with Cobb-Douglas production functions and fixed capital normalized to 1. Thus the production function of firm i in sector j is:

$$y_{it}^j = z_t^j \left(n_{it}^j \right)^{1-\alpha}$$

where capital share α is assumed to be the same across both sectors.

Intermediate good firms are assumed to be subject to a Calvo pricing nominal rigidity. Every period, a random fraction $1 - \phi^j$ of firms in sector j are able to adjust their prices, while the remainder cannot. All firms must hire sufficient labor to meet the demand for

[13]Since this section is fairly standard, I omit many details of the derivation. They can be found in appendix C.

the output they face at their posted price. To remove the inefficiency due to the market power of intermediate goods, I assume that intermediate good firms are provided a production subsidy $1+\tau = \frac{\epsilon}{\epsilon-1}$, proceeds from which are rebated in lump sum payments equally to all firms. Firms discount nominal profits at time t by Q_t. Under these assumptions, firms in sector j that can adjust their price will choose:

$$\left(p_t^{j*}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \frac{1}{1-\alpha}\frac{X_t^j}{Z_t^j}$$

where X_t^j and Z_t^j are defined recursively as

$$X_t^j = w_t \left(p_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left(y_t^j/z_t^j\right)^{\frac{1}{1-\alpha}} + \phi^j \frac{Q_{t+1}}{Q_t} X_{t+1}^j$$
$$Z_t^j = \left(p_t^j\right)^{\epsilon} y_t^j + \phi^j \frac{Q_{t+1}}{Q_t} Z_{t+1}^j$$

Inflation. Suppose that all firms follow this pricing rule. At every point in time, we can distinguish between the aggregate price p_t^j, and the price of adjusting firms p_t^{j*}. The aggregate price level evolves according to:

$$\left(p_t^j\right)^{1-\epsilon} = \phi^j \left(p_{t-1}^j\right)^{1-\epsilon} + \left(1-\phi^j\right)\left(p_t^{j*}\right)^{1-\epsilon}$$

We can write this is inflation terms as:

$$\left(\Pi_t^j\right)^{1-\epsilon} = 1 + \left(1-\phi^j\right)\left[\left(\Pi_t^{j*}\right)^{1-\epsilon} - 1\right]$$

where $\Pi_t^j = p_t^j/p_{t-1}^j$ and $\Pi_t^{j*} = p_t^{j*}/p_{t-1}^j$.

Price Dispersion. In addition to the aggregate price index, we also need to track price dispersion. We would like to define a notion of price dispersion that defines a mapping between aggregate labor supply in sector j, and final output in sector j. The relevant measure of price dispersion is:

$$\Delta_t^j = \int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha}}$$

which implies:

$$y_t^j = z_t^j \left(\frac{n_t^j}{\Delta_t^j} \right)^{1-\alpha}$$

Price dispersion evolves over time according to:

$$\Delta_t^j = \left(\Pi_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left[\phi^j \left(\Delta_{t-1}^j\right) + \left(1-\phi^j\right) \left(\Pi_t^{j*}\right)^{-\frac{\epsilon}{1-\alpha}} \right]$$

3.2 Calibration

The calibration used in the baseline example is shown in Table 1. I calibrate to a quarterly frequency. $\sigma = 1$, $\zeta = 1$ and $\alpha = 1/3$ are standard choices. β is set so that the steady state annual interest rate is 3%. $\delta = 0.025$ gives 10% annual depreciation, which is the average depreciation rate of durable consumer goods (including housing). χ is set to target a durable GDP share of 1/8. The choice of target interest rate, depreciation rate, and durable share are as in Erceg and Levin (2006). ψ is chosen so that steady state labor is 0.8, consistent with the average prime-age employment rate. This allows us to interpret changes in n in terms of changes in the employment rate. $\epsilon = 6$ and $\phi = 2/3$ are as in Gali's textbook; the latter corresponds to prices adjusting every 3 quarters on average, consistent with evidence from micro data. $z^c, z^d = 1$ are normalizations. Finally, ρ is the persistence of the aggregate demand shock θ in the baseline model. This choice implies that demand shocks fade away fairly quickly, and in particular more quickly than prices adjust or the stock of durables depreciate.

Note that this calibration implies that durable goods are just as sticky as nondurable goods. This is a controversial question. Erceg and Levin (2006) also assume identical price stickiness; by contrast, Barsky et al. (2007) argue that housing prices are quite flexible. Álvarez et al. (2006) examine micro price data in the Euro area and conclude that prices of durable and capital goods are stickier than non-durable goods. Since there is some evidence of stickiness across all sectors, and the focus of the present analysis is on differential durability rather than differential price stickiness, I proceed under the assumption of equal price stickiness between the sectors.

σ	α	ζ	δ	β	ϕ	ϵ	χ	ψ	ρ	z^c	z^d
1	$\frac{1}{3}$	1	0.025	0.9926	$\frac{2}{3}$	6	0.18	1.19	0.5	1	1

Table 1: Calibration of Quantitative Example

Interest Elasticities. Given the prominent role played by sectoral interest elasticities in the optimal policy analysis in section 2, one naturally wonders what these elasticities are in the Calvo model. Under the assumption of perfect foresight, we can calculate the equilibrium of the model for a fixed path of interest rates exactly by iterating the equations backward from the steady state. We can then compute the interest elasticities by varying the interest rates at just one point in time. Note that this is different than the more typical approach of computing a monetary policy shock, which considers an innovation to a shock (possibly with persistence) in a Taylor rule. Here I am just marginally adjusting the interest rate at one point in time, and computing the percentage change in other variables in response. This approach is exactly analogous to the interest elasticities computed in section 2 for a marginal change in interest rate.

The resulting interest elasticities for various equilibrium variables are given in Table 2. The two columns correspond to innovations in the current and the future (one period ahead) interest rates respectively. I report elasticities with respect to future interest rates since section 2.4 found that optimal policy under commitment depends on the difference between these elasticities and contemporaneous elasticities.

The results are fairly unsurprising given the model. Nondurable consumption has unit elasticity to both current and future interest rates, which can be seen by iterating the Euler equation forward. Durable consumption is almost twice as sensitive to interest rates as is nondurable consumption; however, given the low depreciation rate of durables, this implies a huge elasticity of 69 for current production of durables y_t^d.[14] By contrast, durable consumption is about half as sensitive to future interest rates as it is to current rates, confirming that the result in Proposition 7 carries through to the case with inflation.

Turning to other variables, the labor elasticity is about 1.5 times the elasticity of total output, which follows mechanically from the assumption of labor share of 2/3. The wage appreciates by a similar magnitude, which is again fairly mechanical given the unit frisch elasticity. Inflation increases marginally, with durable inflation increasing substantially more than nondurable inflation. This results in a relative appreciation of durables of 0.7. Thus over 2/3 of the relative demand effect of the change in interest rates is offset by the immediate increase in price.[15]

[14] This elasticity is quite large compared to empirical estimates, which I review in section 4.2. In appendix A, I discuss adding adjustment costs to the model, which produces more plausible interest elasticities.

[15] The linearized relative demand expression is $\hat{q}_t = \left(\frac{r+\delta}{R}\right)\left(\hat{c}_t - \hat{d}_t\right) + (\hat{q}_{t+1} - \hat{R}_{t+1})\left(\frac{1-\delta}{R}\right)$. Since $(1-\delta)/R \approx 0.97$, a change in relative price affects relative demand about as much as an equivalent change in the interest rate.

	c	d	y^d	n	q	w	Π	Π^c	Π^d
$\epsilon_{R_{t+1}}$	1.00	1.79	69	14.8	0.70	15.8	0.36	0.27	0.97
$\epsilon_{R_{t+2}}$	1.00	0.80	31	7.4	0.65	8.4	0.48	0.40	1.06

Table 2: Elasticities with respect to current and future interest rates.

3.3 Policy Following a Demand Shock

We now examine monetary policy following a demand shock. In particular, I consider a one-time unanticipated 3% decrease in θ at time 0, with θ decaying exponentially with persistence parameter $\rho = 0.5$ thereafter. In a one-sector model with nondurable goods, this shock would be exactly offset by an analogous movement in the interest rate. Thus this shock is equivalent to an exogenous decline in aggregate demand that can be exactly offset by a 3% decline in the interest rate in the first quarter, 1.5% in the second, and so forth. Thus in a one-sector model with either flexible prices or optimal policy this shock would only lower the interest rate.

In a two-sector model, this shock increases the relative demand for durable goods. The impulse responses for the flexible price case are shown in figure 1. The figure shows log deviations of variables from their steady state values. When the interest rate falls, this increases demand for durables, causing some appreciation and some increased production. The overall effect is actually an increase in employment and GDP. This happens because households become more patient following the shock, and thus the economy produces more durables goods to satisfy their desired saving.

Matters are quite different with sticky prices. Now the relative price of durables cannot appreciate as much as in the flexible price equilibrium. Thus cutting the interest rate as much as in the flexible price equilibrium leads to inefficient overproduction of durable goods. I demonstrate this through a comparison of policy rules.

Policy Rules in Aggregate Variables. I first consider policy rules in aggregate variables only. Table 3 summarizes the results of policies in aggregates, from worst to best.[16] The welfare column reports the difference in welfare between the given policy and the flexible price baseline, reported in basis points of consumption. To put these numbers into perspective, the first row reports the welfare of leaving the real interest rate unchanged following the shock, i.e. of simply allowing a 3% decline in demand.

I start with two benchmarks of the literature: inflation targeting and a Taylor rule. The demand shock under inflation targeting is shown in Figure 2. The result produces

[16] Note that the Taylor rule entry gives the weights for the interest rate rule, not for a target that is set to zero under the policy.

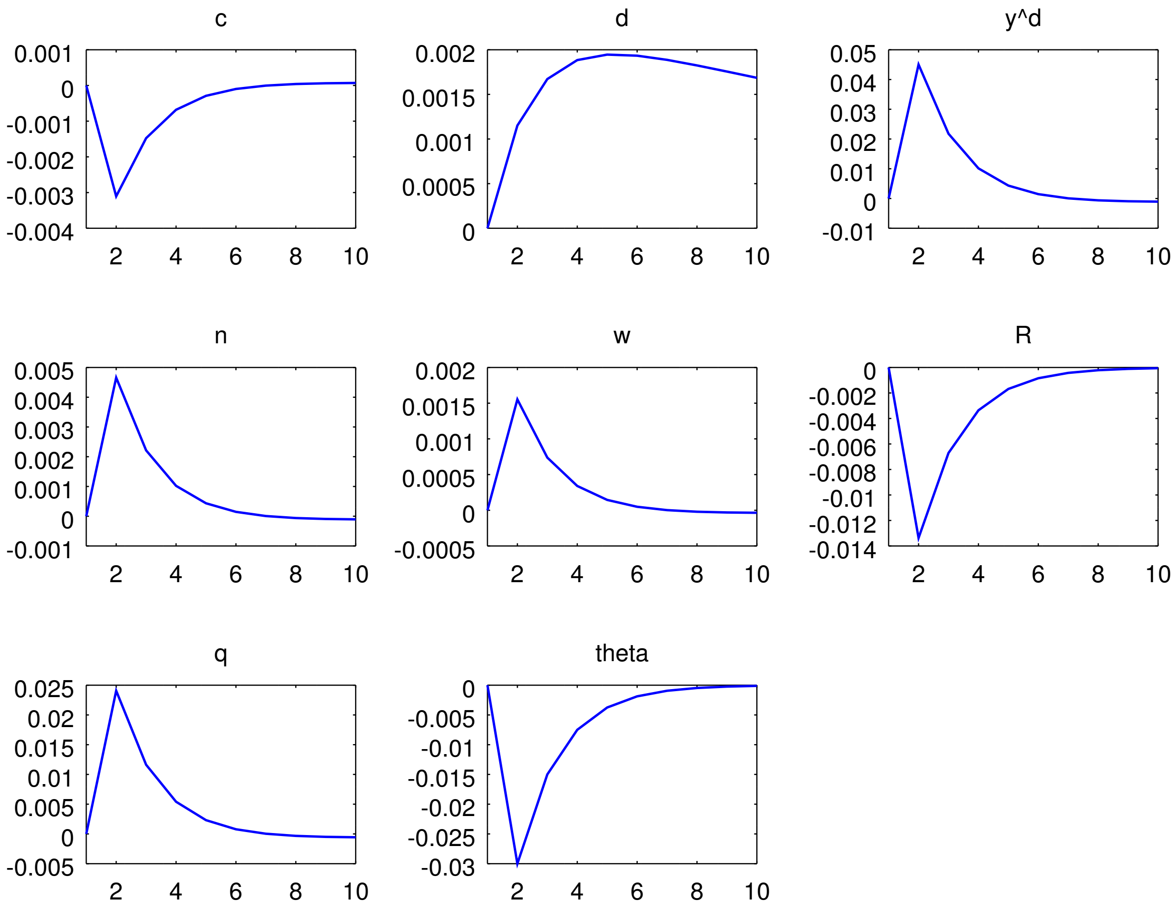

Figure 1: Demand Shock with Flexible Prices

Policy	Π	n	w	Welfare
No Response	0	0	0	-84.7
Nominal Wage	0.5	0	0.5	-19.7
Real Wage	0	0	1	-19.6
Inflation Target	1	0	0	-16.6
Taylor Rule	0.5	0.5	0	-12.8
Labor	0	1	0	-12.2
Optimal Rule	0.518	0.481	0	-10.4

Table 3: Weights and Welfare of Aggregate Policy Rules

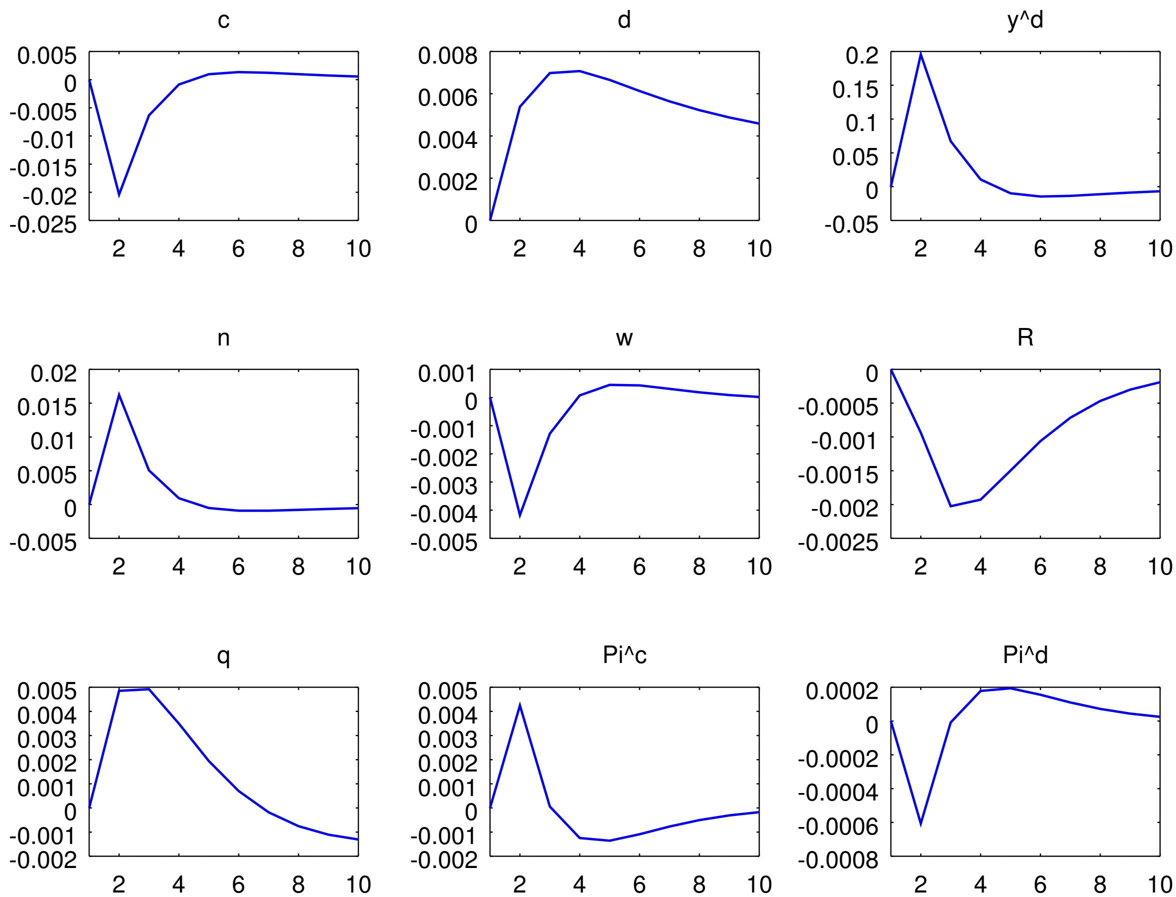

Figure 2: Demand Shock under Inflation Targeting

an increase in aggregate labor as in the flexible price equilibrium, since the interest rate is cut quite substantially. Note that this policy implies substantial sectoral volatility: output of nondurable goods falls by 2% while production of durable goods rises by 20%. This sectoral volatility derives from the sluggish response of the relative price, which appreciates by only 0.5%, compared a to 2.5% appreciation under flexible prices. The result is a welfare loss of about 19.6% of the no-response benchmark. In other words, stabilizing aggregate inflation alone (neglecting sectoral considerations entirely) implies welfare losses about 1/5th as large as ignoring aggregate stabilization and allowing a 3% decline in demand.

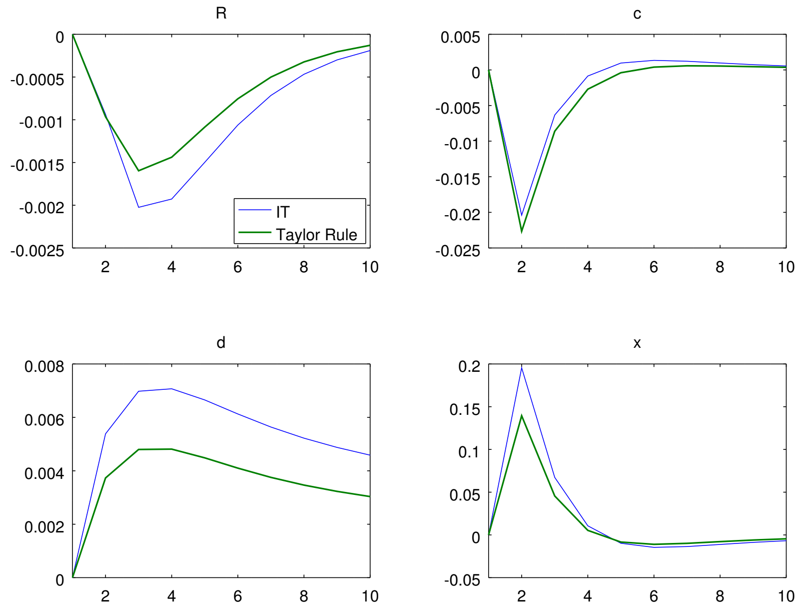

Figure 3: Taylor Rule vs. Inflation Targeting

I next consider a Taylor rule that places places equal weight on inflation and labor, i.e.:

$$\hat{R}_{t+1} = \frac{1}{2}\hat{\Pi}_t + \frac{1}{2}\hat{n}_t$$

Perhaps surprisingly, this simple Taylor rule substantially outperforms inflation targeting in welfare terms. This is because inflation targeting neglects differential sectoral interest sensitivity, and thus cuts interest rates too far. Employment responds to interest rates more than inflation due to the high interest-sensitivity of employment in the durable sector. Thus by placing some weight on labor, the Taylor rule is able to outperform inflation targeting. A comparison of inflation targeting and the Taylor rule is shown in figure 3.

I next consider various other aggregate targets, i.e. targets that may include aggregate wages, labor, and inflation. Targeting a stable real or nominal wage performs poorly — worse than inflation targeting, in fact. By contrast, stabilizing labor is the best of any target that focuses on one variable only, outperforming the Taylor rule marginally. The optimal aggregate target places about equal weight on inflation and labor, and ignores wages. This policy substantially outperforms all other policies in aggregates.

Policy Rules in Sectoral Variables. I next consider policies that may depend on current sectoral variables. The welfare results are reported in table 4.

Policy	Π^d	Π^c	n	w	Welfare
Sectoral Inflation Target	0.54	0.46	0	0	-9.4
Optimal Sectoral Rule	0.68	0.27	0.02	0.02	-9.3

Table 4: Weights and Welfare of Sectoral Policy Rules

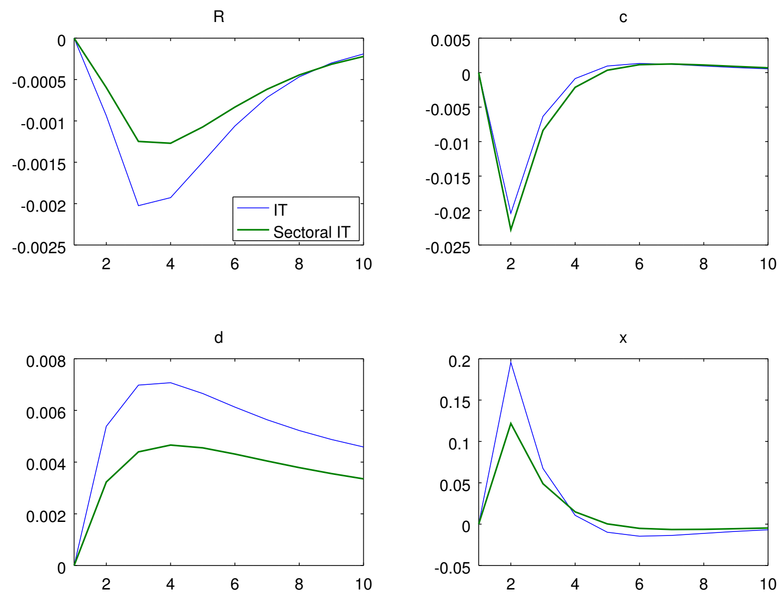

Figure 4: Inflation Target vs. Weighted Inflation Target

First there is weighted sectoral inflation targeting. I find it is optimal to place a weight of 0.54 on durable inflation and 0.46 on nondurable inflation. Thus the optimal target places somewhat greater weight on durables, although they are only 1/8 of GDP. I also consider optimal targets in sectoral inflation, labor, and wages. These differ little from optimal sectoral inflation targeting, as it is optimal to place very little weight on labor and wages.

Figure 4 shows a comparison between regular inflation targeting and optimal sectoral weights. The result is a much smaller response of interest rates to the shock, resulting in lower production in both sectors, but especially in durables. This yields higher welfare due to the tradeoff between aggregate and sectoral stabilization highlighted in section 2.2.1.

Optimal Policy. Finally, I compute the optimal path of interest rates under full commitment. This produces a small but significant gain in welfare relative to weighted sectoral

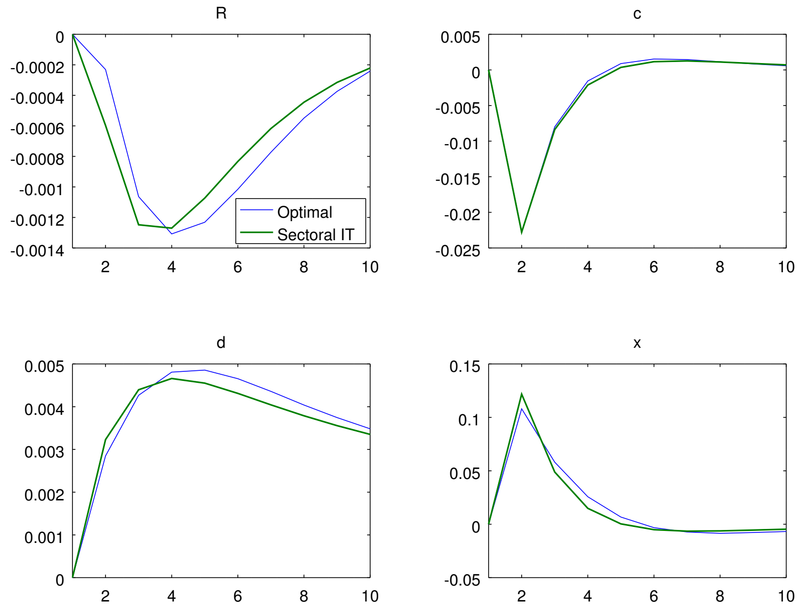

Figure 5: Weighted Inflation Target vs. Optimal Policy

	Π_t^c	Π_t^d	Π_{t-1}^d	Π_{t-2}^d	Π_{t-3}^d	Welfare
Lagged Inflation	0.500	0.499	0.249	0.155	0.080	-8.92
Optimal Policy	–	–	–	–	–	-8.87

Table 5: Optimal Policy vs. Lagged Sectoral Inflation Target

inflation targeting. The welfare gain derives from the use of forward guidance. This becomes clear when we compare the path of interest rates under optimal policy to that under sectoral inflation targeting, which is shown in figure 5. Relative to sectoral inflation targeting, the optimal path of interest rates is shifted into the future, with a muted immediate response and a greater future response. Thus the optimal policy not only places greater weight on stabilizing durable prices, it also preferentially uses forward guidance in the conduct of monetary policy. This is expected given the results of section 2.4.

A Lagged Sectoral Inflation Rule. A natural question at this point is whether there is a simple target that can approximate the optimal path of interest rates. Since the optimal policy seems to improve on sectoral inflation targeting mainly through the use of forward guidance, one way to implement such a policy is to use a *backward-looking* rule, that seeks to stabilize both current and past inflation. That is, one should set interest rates based on lags of inflation.

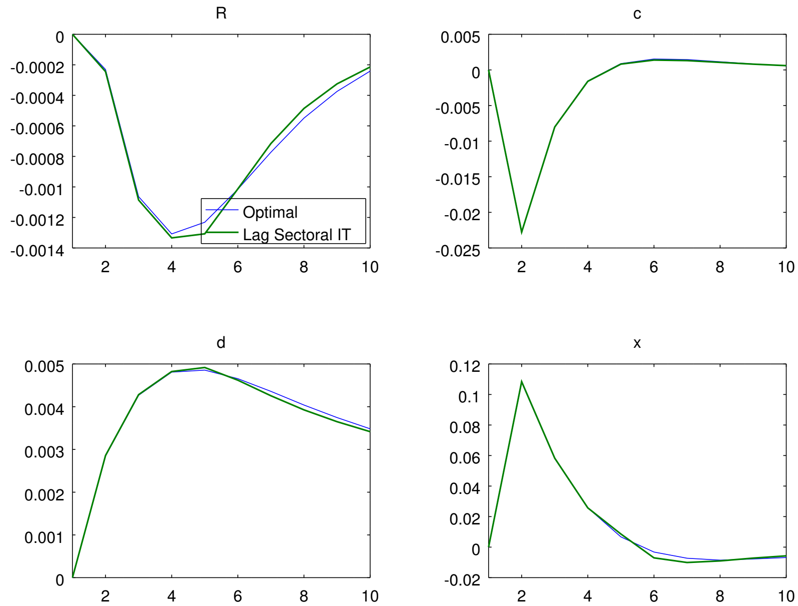

Figure 6: Lag Sectoral Inflation Target vs. Optimal Policy

To test whether such a rule performs well in this model, I search for optimal policy targets that are a linear combination of current and *lagged* sectoral inflation. I allow lags of up to 3 periods. The results are reported in Table 5. The optimal weights on lagged nondurable inflation are zero to significant digits reported in the table. The reported weights are scaled such that the weights on *current* inflation sum to 1.

The optimal weights are surprisingly close to round numbers. I find that it is optimal to place about equal weight on current inflation in each sector, and to weight lags of durable inflation at about half the weight of the previous lag. If this rule carries out to further lags, this suggests that the policymaker should target:

$$\frac{1}{2}\pi_t^d + \frac{1}{2}\pi_t^c + \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k+1}\pi_{t-k}^d = 0$$

Another way to state this is that policy makers should both weight sectors differentially, and also adjust their current inflation target based on past inflation (of the durable sector only!). Altogether, they should place $1/3$ weight on the past history of (durable) inflation. For example, if recent history saw a 1% annual above-target inflation in the durable sector, the optimal policy would call for a 0.5% reduction in the current inflation target (where this target also places equal weights on durable and nondurable goods — in itself a substantial departure from traditional inflation targeting.) Of course, the converse also holds: if the economy has recently experienced a period of durable deflation, it is optimal to raise the inflation target.

Of course, these changes in the inflation target are unlikely to be optimal going forward. This policy requires commitment. The benefit derives not from the current change in the inflation target, but in the expectation of such future changes. This allows the central bank to make greater use of forward guidance in setting policy, and thus utilize current interest rates less. Whether committing to such a policy is feasible is beyond the scope of this paper.

The simple policy performs quite closely to the optimal policy, and a side-by-side comparison is shown in Figure 6. The lag sectoral inflation rule manages to match the optimal initial decline in interest rates nearly exactly, though it is a little off during the recovery period. Nevertheless, the path of interest rates and sectoral output are very close overall, suggesting this policy rule does quite well matching the optimal policy.[17]

4 Monetary Policy in the U.S.

Section 2 derived simple expressions that hold under optimal policy. This section shows how to use these expressions to assess whether policy has achieved a satisfactory degree of sectoral stabilization, and applies this approach to historical U.S. monetary policy. I focus on the static optimality condition:

$$\sum_j \gamma_t^j \epsilon_R^{y_t^j} \tau_t^j = 0$$

In order to apply this expression, we need measures of sectoral labor wedges, sectoral output shares, and sectoral interest elasticities. We must also choose the granularity of the analysis, i.e. how to define a sector. For this exercise, I will stick to a fairly high level of aggregation, using five sectors: services, durable manufactured goods, nondurable manufactured goods, construction, and natural resources (which includes agriculture and mining). The methodology, however, is general and can be applied to a finer granularity if the necessary data is available.

[17] Since the policy rule allowed for three lags only, and the deviation from optimal policy begins in the fourth period, it is possible that including more lags would improve the fit with optimal policy further.

4.1 Sectoral Labor Wedges

Labor wedges are not directly observed in the data, and thus must be inferred. This requires a number of assumptions. Following Gali et al. (2007), and consistent with the functional forms and calibration in section 3, I assume a Cobb-Douglas production function in each sector, and household utility that is separable and isoelastic in labor and in consumption, with unitary Frisch elasticity of labor supply and intertemporal elasticity of substitution. Under these assumptions, the labor wedge in sector j is:

$$\tau_t^j \approx -\widehat{(1-\tau_t^j)} = \hat{p}_t^j + \hat{y}_t^j - \hat{n}_t^j - \hat{n}_t - \hat{c}_t^1 \tag{18}$$

where $\hat{x}$ indicates log deviation of the variable x, and c_t^1 is consumption of the numeraire good.

Computing the labor wedges thus requires data on sectoral real output, relative prices, and employment. For output, I use industry value added as collected by the BEA, and I use employment by industry from the BLS. Unfortunately, quarterly sectoral GDP data is only available since 2005, and thus I use annual data throughout. I focus on private sector output and employment only. Note that value added includes both quantity and relative price movements. I use personal consumption expenditure for the consumption of the numeraire good, and total nonfarm payrolls for aggregate employment.[18] I also compute an aggregate wedge, which uses aggregate GDP and employment. This wedge turns out to be quite close to the weighted sum of labor wedges, weighted by output shares. For each series, I take logs and detrend using a HP-filter with smoothing parameter 6.25, as recommended by Ravn and Uhlig (2002) for annual data. I then compute the labor wedge using equation (18).

The resulting sectoral labor wedges, together with the aggregate labor wedge, are shown in Figure 7 for the period 1960 – 2016. The figure depicts the inverse of the labor wedges, so that the labor wedges are procyclical.[19] The top panel shows the labor wedges for all sectors. The most striking feature is the highly volatile wedge for the natural resources sector. This sector includes both agriculture and mining, including oil and gas extraction, and the high volatility in this sector seems to be driven entirely by oil and gas extraction. However, this sector is fairly small: it averaged 3.2% of (private) GDP over the years 1987 – 2016. To focus on other sectors, particularly the interest-sensitive sectors of construction and durable manufactured goods, the lower panel in figure 7 shows the wedges with natural resources omitted.

[18]More details of the calculations are given in the Appendix.

[19]A positive labor wedge indicates deficient demand, i.e. labor demand below labor supply.

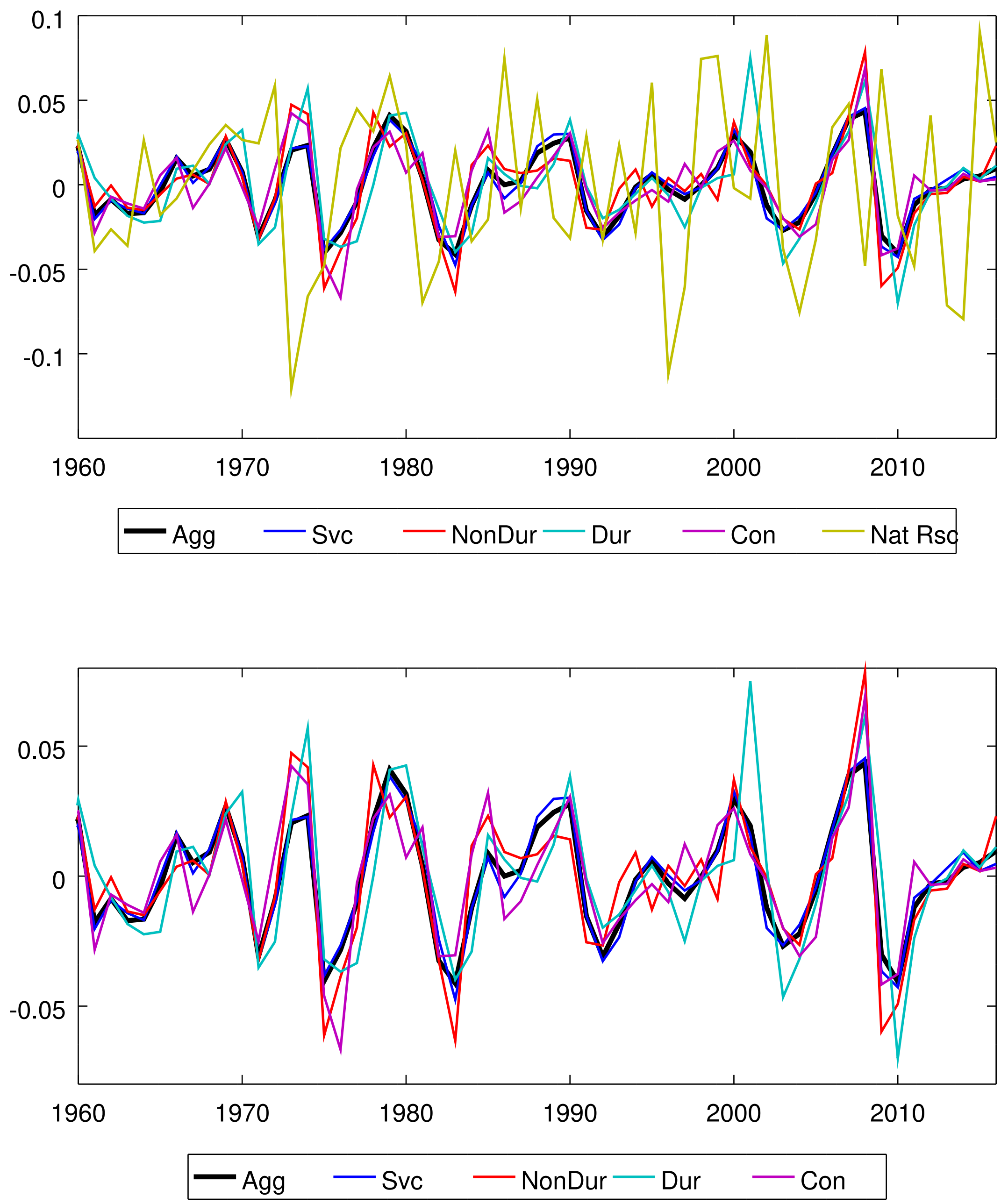

Figure 7: Inverse Sectoral Labor Wedges

Sector	Std. Dev.
Aggregate	0.0215
Services	0.0221
Durables	0.0275
Non-durables	0.0272
Construction	0.0242
Natural Resources	0.0510

Table 6: Standard deviation of sectoral labor wedges

Table 6 reports the standard deviation of each computed labor wedge for the period 1960 – 2016. Unsurprisingly, construction and durable manufactured goods have historically been quite volatile, while services have been less volatile. Perhaps surprisingly, nondurable manufactured goods have been just as volatile as durables, and both are more volatile than construction. Notably, the aggregate labor wedge is less volatile than all sectoral labor wedges, even services.[20] This is consistent with the Fed targeting aggregate output at the expense of sectoral volatility.

Interestingly, the constrution labor wedge does not indicate excessive demand during 2002 – 2006. This is notable given that several commentators have suggested that monetary policy was too loose during this period, which fed the housing bubble.[21] The relatively muted labor wedge in construction during the 2002 – 2006 period suggests that the construction sector was not suffering from excess demand. This is despite large swings in residential investment during this period: between 2002 and 2006, residential investment increased from 5% of GDP to 6.6%, and then declined to 2.5% of GDP by 2010.

How can we reconcile these seemingly contradictory facts? Part of the answer is that construction is not equivalent to residential investment; but even accounting for this a mystery remains. The solution seems to be two-fold: first that the relative price of residential investment is fairly flexible, and second that the capital share of the construction sector is quite low. To see the significance of these two factors, note that we can write the (inverse) labor wedge in sector j as:

$$-\tau_t^j = \left(\frac{\alpha^j}{1-\alpha^j}\right)\hat{y}_t^j - \hat{p}_t^j + A_t$$

where A_t includes aggregate terms, we have assumed no high frequency changes in rela-

[20]This is not an artifact of using aggregate GDP and labor (rather than private) in computing the aggregate labor wedge. The weighted average of the sectoral labor wedges is essentially indistinguishable from the computed aggregate labor wedge, and has a standard deviation of 0.0214.

[21]This argument was perhaps most famously advanced by Taylor (2007).

tive sectoral productivity, and α^j is the capital share of sector j. Valentinyi and Herrendorf (2008) measure the capital shares of various sectors in the U.S., and find that the capital share of construction is about 0.21, compared to the aggregate capital share of 0.33. This implies that a 1% increase in output in the construction sector increases the labor wedge by 1/4%, compared to 1/2% for an average sector. This implies that the construction sector is able to more efficiently accommodate shifts in demand, or equivalently that less relative price movement is necessary to remain at the efficient level of production in this sector.

Sectoral GDP Shares. Sectoral GDP shares are shown in Figure 9 in the appendix. Services are omitted from the figure to highlight the other sectors. There is an overall downward trend in durable and nondurable goods, which is somewhat greater for nondurable goods, and a slight downward trend in the natural resource sector. Construction has remained fairly level over time, though with some fluctuations, e.g. during the housing boom and bust. Services account for the majority of output, and their share has been growing throughout this period as the share of manufactured goods has declined.

4.2 Sectoral Interest Elasticities

To complete the calculation, we need estimates of the interest elasticity in each sector. Unfortunately, there is little consensus in the literature about the magnitudes of these elasticities. In fact, there is a line of empirical research casting doubt on the empirical importance of conventional interest rate channels altogether, whether through cost of capital or intertemporal substitution. For example, Hall (1988) finds "…no strong evidence that the elasticity of intertemporal substitution is positive." Bernanke and Gertler (1995) write that "[E]mpirical studies of supposedly 'interest-sensitive' components of aggregate spending have in fact had great difficulty in identifying a quantitatively important effect of the neoclassical cost-of-capital variable." Mishkin (2007) states that estimates of the interest elasticities of residential investment in the empirical literature "…range from -0.2 to -1.0…In the FRB/US model, used at the Federal Reserve Board, the elasticity is -0.3." More recently, Boivin et al. (2010) state that there are few estimates of durable responses to interest rates, and most estimates are low; that evidence is mixed on the responsiveness of investment, durables, and housing to interest rates; and that estimates of the rate of intertemporal substitution are "modest." Other authors find larger elasticities: Taylor (2007) estimates an interest elasticity of housing starts of 8.3, similar to the value of 8 estimated by Topel and Rosen (1988). While Mankiw (1985) estimates a nondurable interest elasticity of 0.5, he estimates an interest elasticity of 3.4 for the stock of durables, and 13.6 for the flow.

Importantly, what matters for the optimal policy target is not the absolute interest elasticity of each sector, but the *relative* elasticity between sectors. To see this, note that equation (18) is unchanged if we multiply by a constant. On relative sectoral elasticities there is more consistent evidence, which broadly agrees with Mankiw (1985) that durable production is more interest sensitive than nondurables. Bernanke and Gertler (1995) find that, "Nondurables react by much less in percentage terms than durables do." Barsky et al. (2003) find that, following a Romer date, housing starts fall by approximately 33%, residential investment by 22%, automobile sales by 25%, and durables purchases by 12.5%; real GDP does not fall at all, though it declines relative to trend by 6%.[22] Similarly, Boivin et al. (2010) find a limited response of consumption to a monetary policy shock, while there is a large effect on industrial production and housing starts. Erceg and Levin (2006) estimate that "[A] monetary policy shock causes a decline in our broad measure of consumer durables spending that is over three times as large as for the other GDP components."

Given these results, it seems reasonable to conclude that durable goods have a higher interest elasticity than nondurable goods, perhaps 3 times higher. Moreover, there is some evidence that residential investment is still more sensitive to interest rates.

4.3 Assessment of Monetary Policy in the U.S.

Using the sectoral labor wedges, GDP shares, and interest elasticities computed above, we can now use equation (18) to assess the historical conduct of monetary policy in the U.S. For the baseline comparison, I use an interest elasticity of 1 for the services and nondurable goods sectors.[23] Since the labor wedge in the natural resource sector is likely driven mainly by global commodity prices, which are plausibly exogenous to U.S. monetary policy, I omit them from the analysis. Finally, I set the interest elasticity of the durable sector to 3, and of the construction sector to 7.

The result is shown in figure 8. Since the Fed was presumably targeting the aggregate wedge during this period, fluctuations in this wedge likely reflect considerations that we have abstracted from in this analysis. Thus one should focus on the difference between the aggregate and sectoral measures, which indicate whether an aggregate-only monetary

[22]This analysis was presented in the (2003) NBER working paper by this name. A revised version was published as Barsky et al. (2007), but the empirical analysis cited here was not present in the published version.

[23]This should be regarded as a normalization.

policy was overly tight or loose. The overall result seems to be that sectoral considerations should lead the central bank to be more aggressive in stabilizing aggregate output. This occurs because the high interest-elasticity sectors are also more volatile and procyclical than average. Thus placing greater weight on these sectors suggests a more aggressive monetary stabilization policy overall. The bottom panel of figure 8 focuses on the 2000 – 2016 period. Unsurprisingly, given our earlier discussion of the construction wedge, there is no indication that monetary policy was overly loose during 2002 – 2006, at least based on sectoral efficiency considerations.[24]

Given the uncertainty about the sectoral interest elasticities, I conduct a few robustness checks of the analysis. First, I consider a version where the elasticity of construction is 3, the same as durable goods, and find that this leaves the results qualitatively unchanged, although the difference between aggregate and optimal policy targets is reduced somewhat. I also calculate a version including the natural resource sector, with interest elasticity of 2, and find that this does not noticeably alter the results. These results are shown in the appendix in figures 10 and 11.

5 Conclusion

The results of this paper suggest that monetary policymakers should give more consideration to differential sectoral interest sensitivity when setting policy. In particular, policymakers should weight sectors by their interest elasticity, should take dynamic demand effects from durable goods into account when setting policy, and should systematically utilize forward guidance in the conduct of monetary policy.

Several avenues of future work remain. The analysis in this paper highlights the importance of sectoral interest elasticities for monteray policy, yet relatively little empirical work focuses on estimating these elasticities in a consistent way. Further, the model implies that durable goods are relatively more sensitive to current interest rates than to future rates, implying that forward guidance is a useful tool to reduce sectoral volatility; yet I am aware of no work that estimates sectoral elasticities with respect to current vs. future interest rates. Finally, the results imply limitations on the ability of monetary policy alone to stabilize the economy. This suggests that other policies might be useful in supplementing monetary policy, particularly if these policies have sectoral effects that differ from monetary policy.

[24]Of course, this analysis only considers costs of sectoral fluctuations arising from sticky relative prices; it neglects other factors, such as financial stability considerations, that were likely critical in the case of the housing bubble.

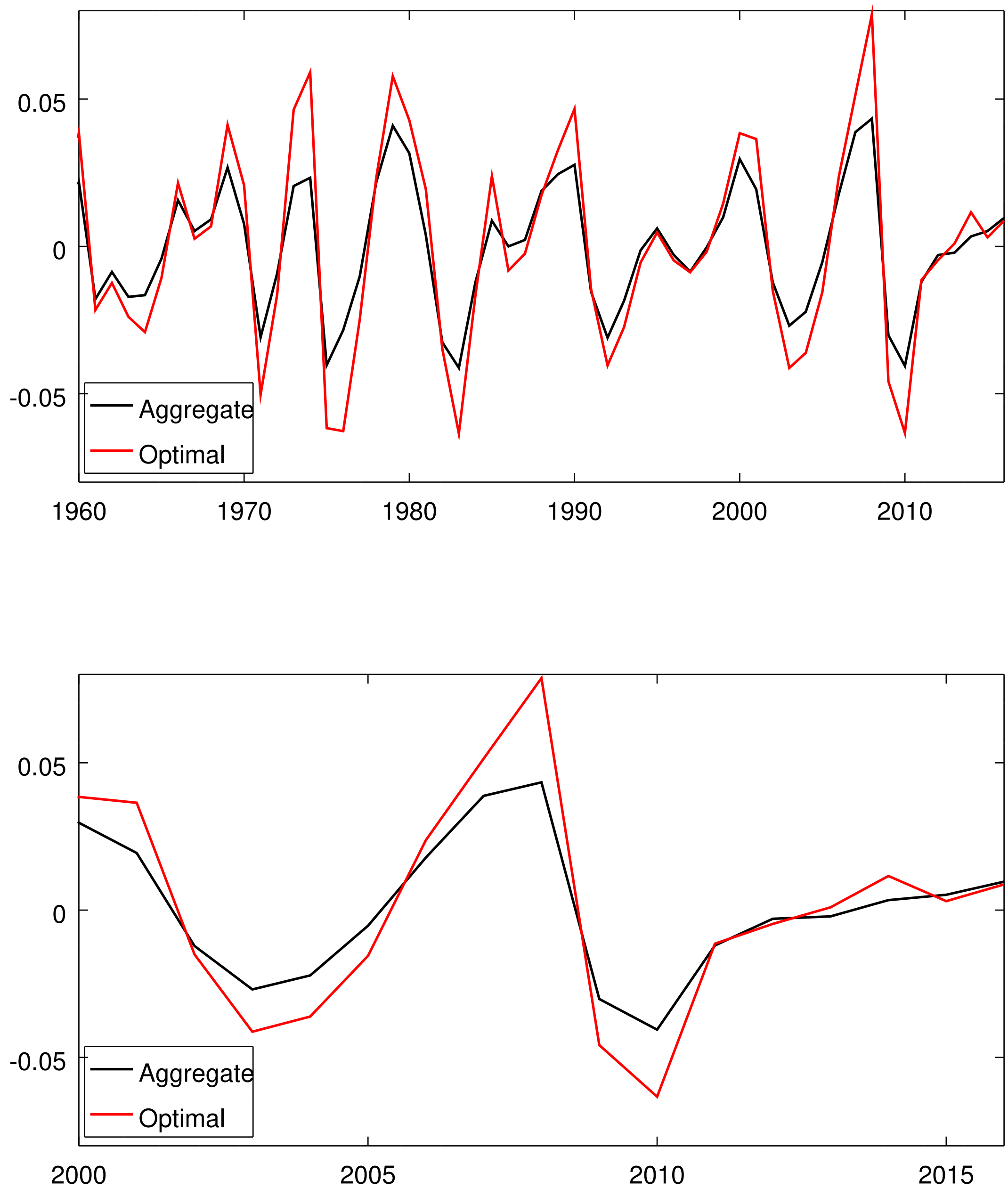

Figure 8: Aggregate vs. optimally-weighted inverse labor wedges

References

Luis J Álvarez, Emmanuel Dhyne, Marco Hoeberichts, Claudia Kwapil, Hervé Le Bihan, Patrick Lünnemann, Fernando Martins, Roberto Sabbatini, Harald Stahl, Philip Vermeulen, et al. Sticky prices in the euro area: A summary of new micro-evidence. *Journal of the European Economic Association*, 4(2-3):575–584, 2006.

Kosuke Aoki. Optimal monetary policy responses to relative-price changes. *Journal of monetary economics*, 48(1):55–80, 2001.

Robert Barsky, Christopher L House, and Miles Kimball. Do flexible durable goods prices undermine sticky price models? Technical report, National Bureau of Economic Research, 2003.

Robert Barsky, Christopher L House, Christoph Boehm, and Miles S Kimball. Monetary policy and durable goods. 2016.

Robert B Barsky, Christopher L House, and Miles S Kimball. Sticky-price models and durable goods. *The American economic review*, 97(3):984–998, 2007.

Paul Beaudry, Dana Galizia, and Franck Portier. Reconciling hayek's and keynes views of recessions. Technical report, National Bureau of Economic Research, 2014.

Pierpaolo Benigno. Optimal monetary policy in a currency area. *Journal of international economics*, 63(2):293–320, 2004.

Ben S Bernanke and Mark Gertler. Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives*, 9(4):27–48, 1995.

Mark Bils, Peter J Klenow, and Benjamin A Malin. Testing for keynesian labor demand. *NBER Macroeconomics Annual*, 27(1):311–349, 2013.

Jean Boivin, Michael T Kiley, Frederic S Mishkin, et al. How has the monetary transmission mechanism evolved over time? *Handbook of Monetary Economics*, 3:369–422, 2010.

Hafedh Bouakez, Emanuela Cardia, and Francisco J Ruge-Murcia. The transmission of monetary policy in a multisector economy. *International Economic Review*, 50(4):1243–1266, 2009.

Hafedh Bouakez, Emanuela Cardia, and Francisco J Ruge-Murcia. Durable goods, intersectoral linkages and monetary policy. *Journal of Economic Dynamics and Control*, 35(5): 730–745, 2011.

Carlos Carvalho et al. Heterogeneity in price stickiness and the real effects of monetary shocks. *The BE Journal of Macroeconomics*, 6(3):1–58, 2006.

Christopher Erceg and Andrew Levin. Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53(7):1341–1359, 2006.

Jordi Gali, Mark Gertler, and J David Lopez-Salido. Markups, gaps, and the welfare costs of business fluctuations. *The review of economics and statistics*, 89(1):44–59, 2007.

Robert E Hall. Intertemporal substitution in consumption. *Journal of political economy*, 96 (2):339–357, 1988.

N Gregory Mankiw. Consumer durables and the real interest rate. *The Review of Economics and Statistics*, pages 353–362, 1985.

N Gregory Mankiw and Ricardo Reis. What measure of inflation should a central bank target? *Journal of the European Economic Association*, 1(5):1058–1086, 2003.

Frederic S Mishkin. Housing and the monetary transmission mechanism. In *Proceedings-Economic Policy Symposium-Jackson Hole*, pages 359–413. Federal Reserve Bank of Kansas City, 2007.

Morten O Ravn and Harald Uhlig. On adjusting the hodrick-prescott filter for the frequency of observations. *The review of economics and statistics*, 84(2):371–376, 2002.

Matthew Rognlie, Andrei Shleifer, and Alp Simsek. Investment hangover and the great recession. Technical report, National Bureau of Economic Research, 2014.

John B Taylor. Housing and monetary policy. Technical report, National Bureau of Economic Research, 2007.

Robert Topel and Sherwin Rosen. Housing investment in the united states. *Journal of Political Economy*, 96(4):718–740, 1988.

Akos Valentinyi and Berthold Herrendorf. Measuring factor income shares at the sectoral level. *Review of Economic Dynamics*, 11(4):820–835, 2008.

A Online Appendix: Durable Adjustment Costs

A potential criticism of the model presented in section 3 is that the interest elasticity of the durable sector is implausibly large.[25] This is particularly concerning given the importance of this elasticity in the optimal policy expressions. To address this objection, I first introduces durable adjustment costs in a general way to the model considered in section 2, and show that the optimal policy expression without commitment is unchanged. I then introduce quadratic adjustment costs to the quantitative model considered in section 3, and show that the qualitative results are unchanged. Although the quantitative significance of sectoral volatility decreases with greater adjustment costs, they remain significant for empirically plausible calibrations.

A.1 Adjustment Costs and Habit Formation in Model without Inflation

Consider the model analyzed in section 2, but with household utility:

$$\sum \beta^t \theta_t \left[u\left(\vec{c}_t, \vec{c}_{t-1}\right) - v\left(n_t\right) \right]$$

This specification nests various common forms of internal habit formation and consumption adjustment costs, as long as they affect the utility function rather than the budget constraint. Household optimality conditions are just as in the baseline model except that the good j pricing equation is:

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \frac{\left(u_{t+1}\right)_{c_t^j}}{R_{t+1}\lambda_{t+1}} + \frac{1-\delta^j}{R_{t+1}} p_{t+1}^j$$

The only difference relative to the baseline case is the inclusion of the term $\left(u_{t+1}\right)_{c_t^j} / \left(R_{t+1}\lambda_{t+1}\right)$ in the good j pricing equation. This term, which will generally be negative, reflects the cost of higher past consumption in the following period. Note that the inclusion of adjustment costs might also affect the value of marginal utility of consumption. Firm optimality conditions and market clearing expressions are just as in the baseline model.

My first result is that the expressions that characterize optimal policy in the one-period fixed price case and the N-period fixed price case without commitment do not change.

Proposition 8 (Optimal Policy without Commitment and Adjustment Costs)**.** *With adjust-*

[25] The baseline calibration implies an interest elasticity of new durable production of 69!

ment costs and N-period fixed prices, the optimal policy without commitment satisfies:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

With $N = 1$*, this becomes:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

Proof. See appendix D. □

Intuitively, the only effect of adjustment costs is to change the magnitudes of the interest elasticities and the precise values of the labor wedges. The static sectoral tradeoff and the effect of durable overhang are fully summarized by the same expressions.

Matters change a bit with commitment. The best we can manage is the following:

Proposition 9 (Optimal Policy with Commitment and Adjustment Costs)**.** *With adjustment costs and N-period fixed prices, the optimal policy with commitment satisfies:*

$$\sum_{t=0}^{N-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_k}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

is the overhang-augmented labor wedge.

Proof. See appendix D. □

This expression is as in Proposition 5 except the sum is up to $t = N - 1$, rather than $t = k$. This is because, with the inclusion of lagged consumption in utility, it is no longer the case that past interest rates have no effect on future consumption. Moreover, we can no longer simplify the expression to apply to two periods only, since the effect of forward guidance is no longer the same for all interest rates after the current period. This is simply because the equations for period t demand depend on $\vec{c}_{t-1}$ and $\vec{c}_{t+1}$, and thus may depend in an arbitrary fashion on all past and future interest rates.

A.2 Quantitative Model with Adjustment Costs

I now add quadratic adjustment costs to the two-sector Calvo model from section 3. Suppose that household utility is:

$$\sum_{t=0}^{\infty} \beta^t \theta_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{d_t^{1-\sigma}}{1-\sigma} - \frac{\varphi}{2} \left(\frac{d_t - d_{t-1}}{d_{t-1}} \right)^2 - \psi \frac{n_t^{1+\zeta}}{1+\zeta} \right]$$

Note that this specification nests the previous model when $\varphi = 0$. Equilibrium conditions are as before except that the durable good pricing equation 16 becomes:

$$q_t = \chi \left(\frac{c_t}{d_t} \right)^{\sigma} - \varphi \left(\frac{d_t - d_{t-1}}{d_{t-1}} \right) \frac{c_t^{\sigma}}{d_{t-1}} + \frac{1}{R_{t+1}} \left[\varphi \left(\frac{d_{t+1} - d_t}{d_t} \right) \frac{d_{t+1} c_{t+1}^{\sigma}}{d_t^2} + (1-\delta)\, q_{t+1} \right] \tag{19}$$

Since the steady state does not change, the parameters from the baseline calibration are still appropriate. The only question is how to calibrate the adjustment cost parameter φ. Rather than take a stand on the correct value for this parameter, I report results for several different values.

Table 7 shows welfare losses for inflation targeting and optimal weighted inflation targeting following a demand shock for various values of φ. Welfare losses are expressed as a percentage of welfare losses in the no response case, with welfare under flexible prices serving as the benchmark. I do this because welfare losses are not directly comparable for different values of φ, since a higher value implies lower welfare losses from relative price stickiness in the first place.[26] I also report the optimal weight on the durable sector and the interest elasticity of new durable production for each value of φ. The interest elasticity shows how the durable adjustment costs affects the volatility of durable demand. The interest elasticity is computed for a 0.1% cut in interest rates.[27]

As expected, adjustment costs reduce the interest elasticity of demand for new durable production by an amount proportional to φ. As implied by the optimal policy results in section 2, this reduces the optimal weight on the durable sector by a commensurate amount. This naturally also reduces the gap between weighting sectors by GDP shares vs. optimal sectoral weights. Moreover, lower relative interest elasticities implies that monetary policy is a better instrument overall, since relative sectoral fluctuations are reduced. This reduces the welfare loss from either policy.

Nevertheless, it is still optimal to place greater weight on the durable sector when

[26]Since adjustment costs allow the marginal cost of durable goods to shift, they effectively substitute for relative price flexibility.

[27]Since the model is nonlinear, a larger cut will prompt a proportionally smaller response in durable demand, since the adjustment cost is larger for larger changes in durable consumption.

φ	$\epsilon_R^{y^d}$	Loss IT	Loss Wght. IT	Dur. Wght.
0	69	19.7%	11.1%	0.54
1	50	19.6%	11.4%	0.53
5	25	16.3%	10.5%	0.48
10	16	12.7%	9.0%	0.44
20	9.1	8.1%	6.4%	0.37
30	6.5	5.5%	4.7%	0.32
50	4.1	3.0%	2.8%	0.26

Table 7: Comparison of Policies for varying levels of φ.

setting monetary policy, resulting in significant welfare gains compared to inflation targeting. For example, consider $\varphi = 20$, which corresponds to a durable interest elasticity of about 9, within the range of plausible empirical estimates.[28] Here it is optimal to weight durable inflation 59% as much as non-durable inflation, compared to 14% under naive inflation targeting. Doing so reduces welfare losses by 21% compared to inflation targeting.

B Online Appendix: Uncertainty

The baseline model in section 2 assumed certainty. This was a convenient assumption to obtain simple optimal policy expressions, but raises the question whether the general results are sensitive to this assumption. This section extends the results to the stochastic case, and shows that the general character of optimal policy does not change.

Flexible Prices. Consider the model in section (2.1) with the inclusion of uncertainty. In particular, I suppose that the demand shock θ is stochastic, and also allow parameters of the production function to be stochastic.[29] I continue to assume that the real bond is safe, so that R_{t+1} is known at time t. Under these assumptions, the household budget constraint is unchanged, and the household objective function becomes:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \theta_t [u\left(\vec{c}_t\right) - v(n_t)]$$

[28]See section 4.2, and recall that what matters here is the relative sectoral elasticity, not the absolute values. Since the baseline model implies an interest elasticity for non-durables of 1, a durable elasticity of 9 corresponds to a ratio of 9:1.

[29]Since the firm production functions given in (6) were time dependent, they already allowed for predictable change in production parameters such as productivity or the shape of the production function. We are now allowing these changeds to be stochastic.

The household optimality expressions are:

$$\frac{v_{n_t}}{\lambda_t} = w_t \tag{20}$$

$$1 = \mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} \frac{\lambda_{t+1}}{\lambda_t} R_{t+1} \right] \tag{21}$$

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + (1-\delta^j)\mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} \frac{\lambda_{t+1}}{\lambda_t} p_{t+1}^j \right] \tag{22}$$

To see what difference uncertainty makes in the demand equations, we combine the good j demand equation with the Euler equation to obtain:

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\theta_{t+1}\lambda_{t+1} p_{t+1}^j \right]}{\mathbb{E}_t \left[\theta_{t+1}\lambda_{t+1} \right]}$$

Under certainty, the term $\theta_{t+1}\lambda_{t+1}$ on the right-hand side cancel out, whereas with uncertainty they do not. Thus there is an additional effect due to covariance between future prices and the future marginal utility of consumption.

One-period fixed prices. We now turn to optimal policy in the case of one-period fixed prices. The optimal policy expression turns out to be just as in the case with certainty:

Proposition 10 (One-period fixed prices with uncertainty)**.** *With one-period fixed prices and uncertainty, the optimal policy is to set the interest rate so that:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

Proof. See appendix D. □

Intuitively, the inclusion of uncertainty makes no difference to optimal policy under one-period fixed prices because optimal tradeoff between sectoral production in the current period is unaffected by uncertainty. Effects of future uncertainty are captured in the equilibrium prices, which are flexible and therefore respond optimally to changes in the covariance arising from changes in current production.

N-period fixed prices without commitment. We next suppose that prices are fixed for multiple periods, and the monetary authority lacks commitment. In this case the policy rule is quite similar to the case with certainty, with the only difference deriving from covariance between future wedges and future marginal utility. The following proposition gives the optimal policy expressions:

Proposition 11 (N-period fixed prices with uncertainty, no commitment)**.** *With N-period fixed prices, no commitment, and uncertainty, the optimal policy is to set the interest rate so that:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\tau_{t+1}^j \theta_{t+1} \lambda_{t+1} \right]}{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \right]}$$

Proof. See appendix D. □

Recall that when $\delta^1 = 1$, so that good 1 is a nondurable good, $\lambda_t = u_{c_t^1}$. This expression differs from the case with certainty by the inclusion of the covariance between the future labor wedge τ_{t+1}^j and the future marginal utility of consumption $\theta_{t+1}\lambda_{t+1}$. A positive covariance in sector j implies that production in this sector is relatively low when aggregate consumption is low, in other words the output gap in this sector is procyclical. This makes the effects of durable overhang greater.

Uncontingent commitment. We next consider cases where the central bank is able to commit to a future interest rates. With commitment, we must make a further distinction: that the central bank can commit to a future path of interest rates, or that it may commit to a future path of *state contingent* interest rates. I start with the former.

Suppose that the monetary authority can commit to a particular path of future interest rates, but cannot commit to a state-contingent rate. This may be because the state is partially unobservable to market participants, and thus only an announced future rate allows the monetary authority to maintain its reputation. Then it is unclear whether the monetary authority will prefer to commit or to retain the flexibility to respond to future shocks. Suppose that the monetary authority commits to a particular path of interest rates over the next N periods. Thus at time 0 the central bank chooses $\{R_{t+1}\}_{t=0}^{N-1}$. The following proposition gives the expression for the optimal choice of R_k:

Proposition 12 (Optimal Policy under Uncontingent Commitment)**.** *In the problem with uncertainty and N-period fixed prices, when the central bank must choose a path of interest rates at time* 0*, the optimal choice of* R_k *satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{k} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_{k+1}}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}$$

is the durable overhang-augmented labor wedge.

Proof. See appendix D. □

As in the certainty case, sectors are equally sensitive to all interest rates more than one period ahead.

Lemma 3 (Symmetric effects of forward guidance under uncertainty). *In the model with uncertainty, $\epsilon_{R_k}^{y_t^j} = \epsilon_{R_\ell}^{y_t^j}$ for $k, \ell \in [t+2, N]$.*

Proof. See appendix D. □

Lemma 3 immediate allows us to obtain an analogous result to Proposition 6.

Corollary 1. *The optimal policy expression in Proposition 12 for $k > 0$ may be written:*

$$\mathbb{E}_0 \frac{y_k}{R_k}\sum_j \gamma_k^j \epsilon_{R_{k+1}}^{y_k^j}\chi_k^j = \mathbb{E}_0 y_{k-1}\sum_j \gamma_{k-1}^j\left(\epsilon_{R_k}^{y_{k-1}^j} - \epsilon_{R_{k+1}}^{y_{k-1}^j}\right)\chi_{k-1}^j$$

Proof. Take the difference between the expressions for the optimal choices of R_{k+1} and R_k in Proposition 12, and apply Lemma 3. □

Compare this expression to the optimal policy expression without commitment, which we can write as:

$$\mathbb{E}_k \sum_j \gamma_k^j \epsilon_{R_{k+1}}^{y_k^j}\chi_k^j = 0$$

The expression with commitment differs in two ways from the no commitment case in two respects. First, the right-hand side contains $\epsilon_{R_k}^{y_{k-1}^j} - \epsilon_{R_{k+1}}^{y_{k-1}^j}$. This captures the potential benefit of using forward guidance, which depends on the differential sensitivity of sectoral volatility to current and future interest rates. Second, the lefthand side contains $\mathbb{E}_0$ rather than $\mathbb{E}_k$. This captures that the monetary authority cannot make the choice of future interest rates state contingent, and thus must commit to future interest rates with the information set available at time 0, rather than at time t.

State-contingent Commitment. Now suppose the central bank can commit to a path of *state-contingent* future interest rates. Let the shock at time t be s_t, let the history of shocks from time 0 to time t by $s^t = \{s_0, \ldots, s_t\}$, and let the probability of this history be $q\left(s^t\right)$. Then at time 0, the central bank chooses:

$$\left\{R_{t+1}\left(s^t\right)\right\}_{t=0}^{N-1}$$

Let $V_N\left(\vec{c}_{N-1}, s^N\right)$ be the flexible price value function at time N. Then the objective function of the central bank is:

$$\sum_{t=0}^{N-1} \sum_{S^t} q\left(s^t\right) \beta^t \theta_t \left[u\left(\vec{c}_t\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right)\right] + \beta^T \sum_{S^T} q\left(s^T\right) \theta_T V\left(\vec{c}_{T-1}\right)$$

where demand $c_t^j\left(s^t\right)$ may depend on the entire path of interest rates $\left\{R_{t+1}\left(s^t\right)\right\}_{t=0}^{N-1}$. Note that Lemma 1 still applies, so that demand at time t does not depend on past choices of interest rates, since current prices are fixed. This demand $c_t^j\left(s^t\right)$ depends only on $\left\{R_{t+k}\left(s^{t+k}\right)\right\}_{k=1}^{N-t}$. The following proposition gives a condition for the optimal choice of $R_{k+1}\left(s_*^k\right)$.

Proposition 13 (Optimal Policy under uncertainty with full commitment)**.** *The optimal choice of $R_{k+1}\left(s_*^k\right)$ in the N-period fixed price model with full commitment is:*

$$\sum_{t=0}^{k} q\left(s_*^t\right) \beta^t \theta_{t*} \lambda_{t*} y_{t*} \left\{\sum_j \epsilon_{R_{k+1}\left(s_*^k\right)}^{y_t^j} \gamma_{t*}^j \chi_{t*}^j\right\} = 0$$

where x_{t*} denotes $x(s_*^t)$, where s_*^t lies on the path defined by s_*^k, i.e. $s_*^k \subset s_*^t$.

Proof. See appendix D. □

To understand these expressions let's analyze the choices of particular interest rates. First the choice of R_1 yields the expression:

$$\sum_j \epsilon_{R_1}^{y_0^j} \gamma_0^j \chi_0^j = 0$$

which is just the same as in the case without commitment, since as before future demand does not depend on past interest rates.[30] Now consider the choice of $R_2\left(s_*^1\right)$. This yields:

$$\theta_0 \lambda_0 y_0 \left\{\sum_j \epsilon_{R_2\left(s_*^1\right)}^{y_0^j} \gamma_0^j \chi_0^j\right\} + q\left(s_*^1\right) \beta \theta_{1*} \lambda_{1*} y_{1*} \left\{\sum_j \epsilon_{R_2\left(s_*^1\right)}^{y_1^j} \gamma_{1*}^j \chi_{1*}^j\right\} = 0$$

[30]That is, Lemma 1 holds for the stochastic case.

Rearranging and taking the difference, we obtain:

$$\sum_j \epsilon^{y_1^j}_{R_2(s_*^1)} \gamma^j_{1*} \chi^j_{1*} = \frac{\theta_0 \lambda_0 y_0}{\beta \theta_{1*} \lambda_{1*} y_{1*}} \left[\sum_j \gamma_0^j \chi_0^j \left(\epsilon^{y_0^j}_{R_1} - \frac{\epsilon^{y_0^j}_{R_2(s_*^1)}}{q\left(s_*^1\right)} \right) \right]$$

This captures the rule of forward guidance. Note that since s_*^1 represents only a subset of states reachable from the initial period, the demand elasticity with respect to the interest rate that prevails in these states will generally be less than the demand elasticity with respect to an interest rate that holds across all states. Dividing the elasticity by $q(s_*^t)$ corrects for this factor, so that the result captures the probability-adjusted interest elasticity of demand. Thus these elasticities should be similar in magnitude, even if the state is quite unlikely to occur.

With contingent interest rates, the symmetric effect of forward guidance at various time horizons no longer holds, and thus we cannot obtain an analogous simplified expression for all future interest rates.

C Online Appendix: Derivation of Calvo Pricing Model

Final Good Aggregator. Suppose that in sector j there is a unit interval of intermediate good firms indexed by i. The output of intermediate good firms is combined by a Dixit-Stiglitz aggregator firm to produce final goods:

$$y_t^j = \left(\int_i \left(y_{it}^j \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

This implies demand function for intermediate goods:

$$\frac{y_{it}^j}{y_t^j} = \left(\frac{p_{it}^j}{p_t^j} \right)^{-\epsilon}$$

Combining these, we obtain an expression for the aggregate price level:

$$\left(p_t^j\right)^{1-\epsilon} = \int_i \left(p_{it}^j\right)^{1-\epsilon}$$

Intermediate Good Firms. Suppose that intermediate good firms produce using production functions:

$$y_{it}^j = z_t^j \left(n_{it}^j\right)^{1-\alpha}$$

Suppose further that firms discount *nominal* profits in time t at rate Q_t. Suppose that there is a per-unit production subsidy $1+\tau = \frac{\epsilon}{\epsilon-1}$, which is chosen to offset under production from firms' market power.

Suppose that every period a random fraction $1-\phi^j$ of firms can adjust their nominal prices, with all other firms leaving their prices unchanged. Consider the price-setting problem of a particular firm, who takes the paths of $\left\{p_t^j, y_t^j, w_t\right\}$ as given, where w_t is the *nominal* wage. Then the price-setting problem of firm i that can adjust its price in period t is to choose p_{it}^j to maximize:

$$\sum_{s=0}^{\infty} \phi^{js} Q_{t+s} \left[(1+\tau) \left(p_{it}^j\right)^{1-\epsilon} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j - w_{t+s} \left(\left(\frac{p_{it}^j}{p_{t+s}^j}\right)^{-\epsilon} \frac{y_{t+s}^j}{z_{t+s}^j} \right)^{\frac{1}{1-\alpha}} \right]$$

Again assuming $1+\tau = \frac{\epsilon}{\epsilon-1}$, this yields optimality condition:

$$\sum_{s=0}^{\infty} \phi^{js} Q_{t+s} \left(p_t^{j*}\right)^{-\epsilon} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j = \frac{1}{1-\alpha} \sum_{s=0}^{\infty} \phi^{js} Q_{t+s} w_{t+s} \left(p_t^{j*}\right)^{-\left(1+\frac{\epsilon}{1-\alpha}\right)} \left(p_{t+s}^j\right)^{\frac{\epsilon}{1-\alpha}} \left(\frac{y_{t+s}^j}{z_{t+s}^j}\right)^{\frac{1}{1-\alpha}}$$

Solving for p_t^{j*}, we obtain:

$$\left(p_t^{j*}\right)^{\left(1+\frac{\alpha\epsilon}{1-\alpha}\right)} = \frac{\sum_{s=0}^{\infty} \phi^{js} Q_{t+s} w_{t+s} \left(p_{t+s}^j\right)^{\frac{\epsilon}{1-\alpha}} \left(\frac{y_{t+s}^j}{z_{t+s}^j}\right)^{\frac{1}{1-\alpha}}}{(1-\alpha) \sum_{s=0}^{\infty} \phi^{js} Q_{t+s} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j}$$

We can express this as:

$$\left(p_t^{j*}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \frac{1}{1-\alpha} \frac{X_t^j}{Z_t^j}$$

where X_t^j and Z_t^j are defined recursively as

$$X_t^j = w_t \left(p_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left(y_t^j / z_t^j\right)^{\frac{1}{1-\alpha}} + \phi^j \frac{Q_{t+1}}{Q_t} X_{t+1}^j$$
$$Z_t^j = \left(p_t^j\right)^{\epsilon} y_t^j + \phi^j \frac{Q_{t+1}}{Q_t} Z_{t+1}^j$$

Aggregate price dynamics. Suppose that all firms follow the pricing rule above. Then at every point in time, we can distinguish between the aggregate price p_t^j, and the price of adjusting firms p_t^{j*}. From the expression for the price index p_t^j, the aggregate price level evolves according to:

$$\left(p_t^j\right)^{1-\epsilon} = \phi^j \left(p_{t-1}^j\right)^{1-\epsilon} + \left(1-\phi^j\right)\left(p_t^{j*}\right)^{1-\epsilon}$$

We can write this is inflation terms as:

$$\left(\Pi_t^j\right)^{1-\epsilon} = 1 + \left(1-\phi^j\right)\left[\left(\Pi_t^{j*}\right)^{1-\epsilon} - 1\right]$$

where $\Pi_t^j = p_t^j / p_{t-1}^j$ and $\Pi_t^{j*} = p_t^{j*} / p_{t-1}^j$.

Price Dispersion. In addition to the aggregate price index, we also need to track price dispersion. We would like a measure price dispersion that relates aggregate sectoral labor demand n_t^j to aggregate sectoral output y_t^j. Aggregate sectoral labor demand satisfies:

$$n_t^j = \int_i n_{it}^j = \int_i \left(\frac{y_{it}^j}{z_t^j}\right)^{\frac{1}{1-\alpha}}$$

Using the intermediate good demand function, we can write this as:

$$n_t^j = \left(\frac{y_t^j}{z_t^j}\right)^{\frac{1}{1-\alpha}} \left[\int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha}}\right]$$

This allows us to define a notion of price dispersion

$$\Delta_t^j = \int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha}}$$

which satisfies:

$$y_t^j = z_t^j \left(\frac{n_t^j}{\Delta_t^j}\right)^{1-\alpha}$$

Dynamics of Price Dispersion. Price dispersion can be written as:

$$\Delta_t^j = \left(p_t^j\right)^{\frac{\epsilon}{1-\alpha}} \cdot \int_i \left(p_{it}^j\right)^{-\frac{\epsilon}{1-\alpha}}$$

Price dispersion evolves over time according to:

$$\Delta_t^j = \left(\Pi_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left[\phi^j \left(\Delta_{t-1}^j\right) + \left(1-\phi^j\right)\left(\Pi_t^{j*}\right)^{-\frac{\epsilon}{1-\alpha}}\right]$$

D Online Appendix: Omitted Proofs

Proof of Proposition 2. The flexible price equilibrium is the unique Pareto Optimum of the economy. Pareto Optimality requires $\tau_t^j = 0$, i.e. $p_t^j = w_t / f_{n_t}^j$. This implies that for every i, j , the relative price between sectors satisfies:

$$\frac{p_t^j}{p_t^i} = \frac{f_{n_t}^j}{f_{n_t}^i}$$

Since $f_{n_t}^j / f_{n_t}^i$ is pinned down by production, this implies that the relative price p_t^j / p_t^i is also pinned down. Due to the normalization $p_t^1 = 1$, this requires that $p_t^j = p_t^{j,flex}$ for all j. This is only feasible if $\bar{p}^j = p_t^{j,flex}$.

If there is linear production in each sector, then Pareto Optimality requires $p_t^j = z_t^j / z_t^i$. Since we start out at a Pareto Optimum, we know this holds for $\bar{p}^j$. Thus, in the absence of idiosyncratic sectoral demand shocks, this expression continues to hold. □

Proof of Proposition 1. The first-order condition of the optimal policy problem is:

$$\sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t} V_{c_t^j}\right) \frac{dc_t^j}{dR} = 0$$

The envelope conditions are:

$$V_{c_t^j} = \left(1 - \delta^j\right) \frac{v_{n_t}}{f_{n_t}^j}$$

Combining these we obtain:

$$\sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t} \left(1 - \delta^j\right) \frac{v_{n_{t+1}}}{f_{n_{t+1}}^j}\right) \frac{dc_t^j}{dR} = 0$$

Next we use that in the next period, since prices are flexible, we have:

$$w_{t+1} = \frac{v_{n_{t+1}}}{u_{c_{t+1}^1}} = p_{t+1}^j f_{n_{t+1}}^j$$

and therefore $\frac{v_{n_{t+1}}}{f^j_{n_{t+1}}} = p^j_{t+1} u_{c^1_{t+1}}$. This may not hold in period t, where instead we have:

$$\frac{v_{n_t}}{f^j_{n_t}} = \left(1 - \tau^j_t\right) p^j_t u_{c^1_t}$$

where τ^j_t is the labor wedge. Therefore optimal policy becomes (after dividing through by $u_{c^1_t}$):

$$\sum_j \left(\frac{u_{c^j_t}}{u_{c^1_t}} + \frac{\beta\theta_{t+1}}{\theta_t} \left(1 - \delta^j\right) p^j_{t+1} \frac{u_{c^1_{t+1}}}{u_{c^1_t}} - \left(1 - \tau^j_t\right) p^j_t \right) \frac{dc^j_t}{dR} = 0$$

Now we substitute in the sector j asset pricing equation to obtain:

$$\sum_j \tau^j_t p^j_t \frac{dc^j_t}{dR} = 0$$

We now write this in terms of interest elasticities of demand and GDP shares. First since $y^j_t = c^j_t - \left(1 - \delta^j\right) c^j_{t-1}$, it follows that $\frac{dy^j_t}{dR} = \frac{dc^j_t}{dR}$. Then we multiply and divide each term of the sum by y^j_t to put things in terms of production, multiply the entire expression through by $-R$, and then divide through by GDP, which is $y_t = \sum_j p^j_t y^j_t$. Then the expression can be written as:

$$\sum_j \epsilon^j_R \gamma^j_t \tau^j_t = 0$$

where $\epsilon^j_R = -\frac{dy^j_t}{dR_{t+1}} \frac{R_{t+1}}{y^j_t}$, $\gamma^j_t = \frac{p^j_t y^j_t}{\sum_j p^j_t y^j_t}$, and $\tau^j_t = 1 - \frac{w_t}{p^j_t f^j_{n_{t+1}}}$. □

Proof of Proposition 3. Let ϵ and τ be random variables which take on values $\left(\epsilon^j, \tau^j\right)$ in state j, which occurs with probability γ^j. Then $\epsilon^y = E[\epsilon]$, $\tau^y = E[\tau]$, and $\sum_j \gamma^j \left(\epsilon^j_R - \epsilon^y_R\right) \left(\tau^j - \tau^y\right) = \text{Cov}(\epsilon, \tau) = E[\epsilon\tau] - E[\epsilon] E[\tau]$. The static policy rule is then $E[\epsilon\tau] = \sum_j \gamma^j \epsilon^j_R \tau^j = 0$, and therefore under optimal policy we have $E[\epsilon] E[\tau] + \text{Cov}(\epsilon, \tau) = 0$. Dividing through by ϵ^y_R then yields the result. □

Proof of Proposition 4. The optimality expression is just as in the case with one-period fixed prices:

$$\sum_j \left(u_{c^j_t} - \frac{v_{n_t}}{f^j_{n_t}} + \frac{\beta\theta_{t+1}}{\theta_t} \left(1 - \delta^j\right) \frac{v_{n_{t+1}}}{f^j_{n_{t+1}}} \right) \frac{dc^j_t}{dR} = 0$$

But now it may be the case that $\tau^j_{t+1} \neq 0$. Therefore we must use the expression:

$$\frac{v_{n_t}}{f^j_{n_t}} = \left(1 - \tau^j_t\right) p^j_t u_{c^1_t}$$

in both period t and $t+1$. Using this, the expression above becomes:

$$\sum_j \left(\frac{u_{c^j_t}}{u_{c^1_t}} + \left(\frac{1-\delta^j}{R_{t+1}}\right) \left(1 - \tau^j_{t+1}\right) p^j_{t+1} - \left(1 - \tau^j_t\right) p^j_t \right) \frac{dc^j_t}{dR} = 0$$

Now we use the expression for p^j_t, together with the fact that $p^j_t = p^j_{t+1} = \bar{p}^j$, to obtain:

$$\sum_j \bar{p}^j \left(\tau^j_t - \left(\frac{1-\delta^j}{R_{t+1}}\right) \tau^j_{t+1} \right) \frac{dc^j_t}{dR} = 0$$

Now, as before, we note that $\frac{dy^j_t}{dR} = \frac{dc^j_t}{dR}$, we multiply and divide each term of the sum by y^j_t, multiply the entire expression by $-R$, and then divide through by GDP $y_t = \sum_j p^j_t y^j_t$, to obtain:

$$\sum_j \epsilon^j_R \gamma^j_t \tau^j_t = \sum_j \epsilon^j_R \gamma^j_t \left(\frac{1-\delta^j}{R_{t+1}}\right) \tau^j_{t+1}$$

□

Proof of Lemma 1. We show this by backward induction. First consider the expressions in period $T = N-1$, i.e. the last period with fixed prices. Here the expressions as:

$$\frac{u_{c^j_T}\left(\vec{c}_T\right)}{u_{c^1_T}\left(\vec{c}_T\right)} = \bar{p}^j - \frac{1-\delta^j}{R_{T+1}} p^j_{T+1}\left(\vec{c}_T\right)$$
$$\frac{u_{c^1_T}\left(\vec{c}_T\right)}{u_{c^1_{T+1}}\left(\vec{c}_T\right)} = \frac{\beta\theta_{T+1}}{\theta_T} R_{T+1}$$

where current marginal utility is a function of current consumption only from the assumption of time-separability of utility, and where future prices and consumption depend only on current consumption because these are defined by the flexible price equilibrium for given initial state, which here is just $\vec{c}_T$. Note that we have N_j equations in N_j unknowns, given entirely in terms of $\left(\vec{c}_T, R_{T+1}, \bar{p}^j\right)$. Thus these expressions implicitly define current demand as a function of current fixed prices and the current interest rate only:

$$\vec{c}_T\left(R_{T+1}\bar{p}^j\right)$$

Now we show by induction that demand equations for earlier periods depend only on future interest rates. Suppose this is true for all future periods up to period T. Then in period t we have:

$$\frac{u_{c_t^j}\left(\vec{c}_t\right)}{u_{c_t^1}\left(\vec{c}_t\right)} = \bar{p}^j\left(1-\frac{1-\delta^j}{R_{t+1}}\right)$$
$$\frac{u_{c_t^1}\left(\vec{c}_t\right)}{u_{c_{t+1}^1}\left(\{R_s\}_{s\geq t+2}\right)} = \frac{\beta\theta_{t+1}}{\theta_t}R_{t+1}$$

These are again N_j equations in N_j unknowns, only in terms of $\left(\vec{c}_t, \vec{\bar{p}}, \{R_s\}_{s\geq t+1}\right)$. Therefore these expressions implicitly define $\vec{c}_t\left(\vec{\bar{p}}, \{R_s\}_{s\geq t+1}\right)$. □

Proof of Proposition 5. Consider the choice of R_k for $k \in \{1,\ldots,N\}$. The optimality condition is:

$$\sum_j\left[\sum_{t=0}^{N-2}\frac{\beta^t\theta_t}{\theta_0}\left(\frac{\partial U_t}{\partial c_t^j}+\frac{\beta\theta_{t+1}}{\theta_t}\frac{\partial U_{t+1}}{\partial c_t^j}\right)\frac{dc_t^j}{dR_k}\right]=0$$
$$+\sum_j\left[\frac{\beta^{N-1}\theta_{N-1}}{\theta_0}\left(\frac{\partial U_{N-1}}{\partial c_{N-1}^j}+\frac{\beta\theta_N}{\theta_{N-1}}\frac{dV_N}{dc_{N-1}^j}\right)\frac{dc_{N-1}^j}{R_k}\right]=0$$

Now we use the fact that:

$$U_{c_t^j} = u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} = u_{c_t^1}\left[\frac{u_{c_t^j}}{u_{c_t^1}} - \left(1-\tau_t^j\right)p_t^j\right]$$
$$U_{c_{t-1}^j} = \left(1-\tau_t^j\right)\left(1-\delta^j\right)u_{c_t^1}p_t^j$$

Then the optimality condition becomes:

$$\sum_j\left[\sum_{t=0}^{k-1}\beta^t\lambda_t\bar{p}^j\tau_t^j\left(1-\left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\tau_{t+1}^j}{\tau_t^j}\right)\frac{dc_t^j}{dR_k}\right]=0$$

Note that this holds for every $R_k \in \{R_1,\ldots,R_N\}$. We can then write this as:

$$\sum_{t=0}^{k-1}\beta^t\lambda_t y_t\left[\sum_j\left(\gamma_t^j\tau_t^j-\gamma_t^j\tau_{t+1}^j\left(\frac{1-\delta^j}{R_{t+1}}\right)\right)\epsilon_{R_k}^{y_t^j}\right]=0$$

□

Proof of Lemma 2. Demand for c_t^j for $j \neq 1$ is defined by the set of equations:

$$
\begin{aligned}
u_{c_t^j} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j u_{c_t^1} \\
u_{c_t^1} &= \frac{\beta\theta_{t+1}}{\theta_t} R_{t+1} u_{c_{t+1}^1}
\end{aligned}
$$

Iterating the Euler equation forward in time, we obtain:

$$
u_{c_t^1} = \beta^{N-t} \frac{\theta_N}{\theta_t} \left(\prod_{s=t+1}^{N} R_s \right) u_{c_N^1}
$$

Substituting this into the system of equations above yields:

$$
\begin{aligned}
u_{c_t^j} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j u_{c_t^1} \\
u_{c_t^1} &= \beta^{N-t} \frac{\theta_N}{\theta_t} \left(\prod_{s=t+1}^{N} R_s \right) u_{c_N^1}
\end{aligned}
$$

Since there are N_j of these equations and N_j unknowns, this set of equations determines $\vec{c}_t \left(\{R_s\}_{s=t+1}^{N}, \bar{p}^j, \theta_t, \theta_N, u_{c_N^1} \right)$. Now consider R_k for $k \in [t+2, N]$. By Lemma 1, $u_{c_N^1}$ is not a function of R_k. Then if we differentiate the system of equations above by R_k, we obtain:

$$
\begin{aligned}
\sum_i u_{c_t^j c_t^i} \frac{dc_t^i}{dR_k} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j \left(\sum_i u_{c_t^1 c_t^i} \frac{dc_t^i}{dR_k} \right) \\
\sum_i u_{c_t^1 c_t^i} \frac{dc_t^i}{dR_k} &= \frac{u_{c_t^1}}{R_k}
\end{aligned}
$$

We can write this in terms of consumption demand interest elasticities as:

$$
\begin{aligned}
\sum_i u_{c_t^j c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j \left(\sum_i u_{c_t^1 c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} \right) \\
\sum_i u_{c_t^1 c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} &= -u_{c_t^1}
\end{aligned}
$$

This yields N_j equations in N_j unknowns, namely the interest elasticities $\epsilon_{R_k}^{c_t^i}$. But note that these expressions are the same for any $k \in [t+2, N]$, and thus the elasticities are the same. This proves the proposition. □

Proof of Proposition 6. We can write the optimality expressions as:

$$\beta^{k-1}\lambda_{k-1}y_{k-1}\left[\sum_j\left(\gamma^j_{k-1}\tau^j_{k-1}-\gamma^j_{k-1}\tau^j_k\left(\frac{1-\delta^j}{R_{k1}}\right)\right)\epsilon^{y^j_{k-1}}_{R_k}\right] = -\sum_{t=0}^{k-2}\beta^t\lambda_t y_t\left[\sum_j\left(\gamma^j_t\tau^j_t-\gamma^j_t\tau^j_{t+1}\left(\frac{1-\delta^j}{R_{t+1}}\right)\right)\right.$$

Take the difference:

$$\beta^{k-1}\lambda_{k-1}y_{k-1}\left[\sum_j\left(\gamma^j_{k-1}\tau^j_{k-1}-\gamma^j_{k-1}\tau^j_k\left(\frac{1-\delta^j}{R_{k1}}\right)\right)\epsilon^{y^j_{k-1}}_{R_k}\right] = \sum_{t=0}^{k-2}\beta^t\lambda_t y_t\left[\sum_j\left(\gamma^j_t\tau^j_t-\gamma^j_t\tau^j_{t+1}\left(\frac{1-\delta^j}{R_{t+1}}\right)\right)\left(\epsilon\right.\right.$$

The right-hand terms cancel for $t < k-2$. Then we obain:

$$\sum_j\gamma^j_{k-1}\left(\tau^j_{k-1}-\tau^j_k\left(\frac{1-\delta^j}{R_{k1}}\right)\right)\epsilon^{y^j_{k-1}}_{R_k} = \frac{R_{k-1}y_{k-2}}{y_{k-1}}\left[\sum_j\gamma^j_{k-2}\left(\tau^j_{k-2}-\tau^j_{k-1}\left(\frac{1-\delta^j}{R_{k-1}}\right)\right)\left(\epsilon^{y^j_{k-2}}_{R_{k-1}}-\epsilon^{y^j_{k-2}}_{R_k}\right)\right]$$

□

Proof of Proposition 7. The demand equations can be written:

$$u_{c^j_t} = \left(1-\frac{1-\delta^j}{R_1}\right)\bar{p}^j u_{c^1_t}$$
$$u_{c^1_t} = \beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$

where the second equation is the Euler equation iterated forward an extra period. Combining these, we obtain:

$$u_{c^j_t} = \left(1-\frac{1-\delta^j}{R_1}\right)\bar{p}^j\beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$
$$u_{c^1_t} = \beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$

Since (by Lemma 1) c^j_{t+2} does not depend on R_{t+1} or R_{t+2}, we can immediately compute the following:

$$u_{c^j_t c^j_t}\frac{dc^j_t}{dR_{t+2}} = \frac{u_{c^j_t}}{R_{t+2}}$$
$$u_{c^j_t c^j_t}\frac{dc^j_0}{dR_{t+1}} = \frac{u_{c^j_t}}{r_{t+1}+\delta^j}$$

We can express this as:

$$\sigma_t^j \epsilon_{R_{t+2}}^{y_t^j} \frac{y_t^j}{c_t^j} = 1$$
$$\sigma_t^j \epsilon_{R_{t+1}}^{y_0^j} \frac{y_t^j}{c_t^j} = \frac{1 + r_{t+1}}{r_{t+1} + \delta^j}$$

where $\sigma_t^j = -\frac{u_{c_t^j c_t^j} c_t^j}{u_{c_t^j}}$. Or in other words:

$$\epsilon_{R_{t+2}}^{y_t^j} = \left(\frac{r_{t+1} + \delta^j}{1 + r_{t+1}}\right) \epsilon_{R_{t+1}}^{y_t^j}$$

Taking the difference, this implies:

$$\epsilon_{R_{t+1}}^{y_t^j} - \epsilon_{R_{t+2}}^{y_t^j} = \left(\frac{1 - \delta^j}{R_{t+1}}\right) \epsilon_{R_{t+1}}^{y_t^j}$$

Plugging this into the optimal policy expression found in 5 yields the last expression. □

Proof of Proposition 10. As before, labor can be expressed as a function of demand as:

$$n_t\left(\vec{c}_t\right) = \sum_j \left(f_t^j\right)^{-1} \left(c_t^j - \left(1 - \delta^j\right) c_{t-1}^j\right)$$

Then the optimal policy problem is to choose R_{t+1} to maximize:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right)\right] \right\}$$

The optimality expression is:

$$\sum_j \left(u_{c_t^j} + \mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t} V_{c_t^j}\right] - \frac{v_{n_t}}{f_{n_t}^j}\right) \frac{dc_t^j}{dR} = 0$$

From the envelope condition, we have:

$$V_{c_{t-1}^j} = \left(1 - \delta^j\right) \frac{v_{n_t}}{f_{n_t}^j}$$

But since the next period has flexible prices, this implies:

$$V_{c_t^j} = \left(1-\delta^j\right) p_{t+1}^j \lambda_{t+1}$$

Substituting this into the expression above, we obtain:

$$\sum_j \left(u_{c_t^j} + \mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} \left(1-\delta^j\right) p_{t+1}^j \lambda_{t+1} \right] - \frac{v_{n_t}}{f_{n_t}^j} \right) \frac{dc_t^j}{dR} = 0$$

This we may write this as:

$$\sum_j \lambda_t p_t^j \left(1 - \frac{w_t}{p_t^j f_{n_t}^j} \right) \frac{dc_t^j}{dR} = 0$$

which, as before, we may write as:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

□

Proof of Proposition 11. We again write the optimal policy problem as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right) \right] \right\}$$

The optimality expression is again:

$$\sum_j \left(u_{c_t^j} + \mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} V_{c_t^j} \right] - \frac{v_{n_t}}{f_{n_t}^j} \right) \frac{dc_t^j}{dR} = 0$$

But now it may be the case that $\tau_{t+1}^j \neq 0$. Thus we must use the expression:

$$V_{c_t^j} = \left(1-\delta^j\right) \frac{v_{n_{t+1}}}{f_{n_{t+1}}^j} = \left(1-\delta^j\right) \left(1-\tau_{t+1}^j\right) p_{t+1}^j \lambda_{t+1}$$

Using this, the expression above becomes:

$$\sum_j \left(u_{c_t^j} + \left(1-\delta^j\right) \mathbb{E}_t \left[\frac{\beta\theta_{t+1}}{\theta_t} \left(1-\tau_{t+1}^j\right) p_{t+1}^j \lambda_{t+1} \right] - \frac{v_{n_t}}{f_{n_t}^j} \right) \frac{dc_t^j}{dR} = 0$$

Now we use the expression for p_t^j, together with the fact that $p_t^j = p_{t+1}^j = \bar{p}^j$, and the definition of R_{t+1} to obtain:

$$\sum_j \bar{p}^j \left(\tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \tau_{t+1}^j \right]}{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \right]} \right) \frac{dc_t^j}{dR} = 0$$

Now, as before, we note that $\frac{dy_t^j}{dR} = \frac{dc_t^j}{dR}$, we multiply and divide each term of the sum by y_t^j, multiply the entire expression by $-R$, and then divide through by GDP $y_t = \sum_j p_t^j y_t^j$, to obtain:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\theta_{t+1} \tau_{t+1}^j \lambda_{t+1} \right]}{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \right]}$$

□

Proof of Proposition 12. The problem of the monetary authority is:

$$\max_{\{R_{t+1}\}_{t=0}^{N-1}} \sum_{t=0}^{N-1} \sum_{s^t} q\left(s^t\right) \beta^t \theta_t \left[u\left(\vec{c}_t\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right) \right] + \beta^T \sum_{S^T} q\left(s^T\right) \theta_T V\left(\vec{c}_{T-1}\right)$$

As before, past choices of interest rates do not affect future equilibrium variables. Thus the optimality condition for the choice of R_{k+1} is:

$$\sum_{t=0}^{k} \sum_{s^t} q\left(s^t\right) \beta^t \theta_t \sum_j \left[\left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t^j}} \right) \left(c_t^j \right)_{R_{k+1}} \right] + \sum_{t=1}^{k+1} \sum_{s^t} q\left(s^t\right) \beta^t \theta_t \sum_j \left(1-\delta^j\right) \frac{v_{n_t}}{f_{n_t^j}} \left(c_{t-1}^j \right)_{R_{k+1}} = 0$$

We reindex to write these as:

$$\sum_{t=0}^{k} \sum_{s^t} q\left(s^t\right) \beta^t \theta_t \sum_j \left[u_{c_t^j} - \frac{v_{n_t}}{f_{n_t^j}} + \beta \left(1-\delta^j\right) \sum_{s^{t+1} \subset s^t} \frac{q\left(s^{t+1}\right)}{q\left(s^t\right)} \frac{\theta_{t+1}}{\theta_t} \frac{v_{n_{t+1}}}{f_{n_{t+1}^j}} \right] \left(c_t^j \right)_{R_{k+1}} = 0$$

Using $v_{n_t}/f_{n_t^j} = \left(1-\tau_t^j\right) \lambda_t p_t^j$ and $u_{c_t^j} = p_t^j \lambda_t - \beta\left(1-\delta^j\right) \mathbb{E}_t \frac{\theta_{t+1}}{\theta_t} \lambda_{t+1} p_{t+1}^j$, we can write this as:

$$\sum_{t=0}^{k} \sum_{s^t} q\left(s^t\right) \beta^t \theta_t \sum_j \left[\tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\sum_{s^{t+1} \subset s^t} q\left(s^{t+1}\right) \theta_{t+1} \lambda_{t+1} \tau_{t+1}^j}{\sum_{s^{t+1} \subset s^t} q\left(s^{t+1}\right) \theta_{t+1} \lambda_{t+1}} \right] \lambda_t p^j \left(c_t^j \right)_{R_{k+1}} = 0$$

or just:

$$\mathbb{E}_0 \sum_{t=0}^{k} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_{k+1}}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \tau_{t+1}^j \right]}{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \right]}$$

is the durable overhang-augmented labor wedge. □

Proof of Lemma 3 . As in the certainty case, we can iterate the Euler equation forward to show that the effect of the interest more than 1 period ahead is the same. First observe that the good j demand equation at time $t < N-1$ can be written as:

$$u_{c_t^j} = \left(1 - \frac{1-\delta^j}{R_{t+1}} \right) p^j \lambda_t$$

Since p^j is fixed, the only moving parts here are R_{t+1} and λ_t. By iterating the Euler equation forward, we obtain:

$$\theta_t \lambda_t = \beta^{N-t} \left(\prod_{k=t}^{N-1} R_{k+1} \right) \mathbb{E}_t \left[\theta_N \lambda_N \right]$$

Note that R_{k+1} for $k \in [t+1, N-1]$ has exactly the same effect on demand. □

Proof of Proposition 13. Start with the objective function

$$\sum_{t=0}^{N-1} \sum_{S^t} q\left(s^t\right) \beta^t \theta_t \left[u\left(\vec{c}_t\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right) \right] + \beta^T \sum_{S^T} q\left(s^T\right) \theta_T V\left(\vec{c}_{T-1}\right)$$

The first-order condition with respect to $R_{k+1}\left(s_*^k\right)$ satisfies:

$$\sum_{t=0}^{k} q\left(s_*^t\right) \beta^t \theta_t \sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} \right) \frac{dc_t^j}{dR_{k+1}\left(s_*^k\right)} + \sum_{t=0}^{k} \sum_{s^{t+1} \subset s_*^t} q\left(s^{t+1}\right) \beta^{t+1} \theta_{t+1} \sum_j \left[\left(1-\delta^j\right) \frac{v_{n_{t+1}}}{f_{n_{t+1}}^j} \frac{dc_t^j}{dR_{k+1}\left(s_*^k\right)} \right] = 0$$

where the states s_*^t are taken to lie on the path defined by s_*^k, that is $s_*^k \subset s_*^t$. Note that since the choice of $R_{k+1}\left(s_*^k\right)$ may depend on the entire history of past states, its effect on c_t^j only occurs in the state s_*^t, i.e. the state that yields directly to s_*^k. However, this increase in $c_t^j\left(s_*^t\right)$ will lead to an increase in the initial stock of goods $c_t^j\left(s^{t+1}\right)$ in all state s^{t+1} reachable from s_*^t, including ones not leading to s_*^k. Now we use the fact that $u_{c_t^j} = \bar{p}^j\lambda_t\left(1-\frac{1-\delta^j}{R_{t+1}}\right)$, $\frac{v_n}{f_n^j} = \left(1-\tau_t^j\right)\bar{p}^j\lambda_t$, and

$$\frac{1}{R_{t+1}\left(s_*^t\right)} = \mathbb{E}_{t*}\left[\beta\frac{\theta_{t+1}}{\theta_t}\frac{\lambda_{t+1}}{\lambda_t}\right] = \sum_{s^{t+1}\subset s_*^t}\left[\frac{q\left(s^{t+1}\right)}{q\left(s_*^t\right)}\beta\frac{\theta_{t+1}}{\theta_t}\lambda_{t+1}\right]$$

to obtain:

$$\sum_{t=0}^{k} q\left(s_*^t\right)\beta^t\theta_t\lambda_t y_t\left\{\sum_j \epsilon_{R_{k+1}\left(s_*^k\right)}^{y_t^j}\gamma_t^j\left(\tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}\left(s_*^t\right)}\right)\frac{\mathbb{E}_{t*}\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_{t*}\left[\lambda_{t+1}\theta_{t+1}\right]}\right)\right\} = 0$$

□

Proof of Proposition 8. Consider the N-period fixed price case without commitment. We write the problem of the policymaker as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_{R_{t+1}}\left\{u\left(\vec{c}_t,\vec{c}_{t-1}\right) - v\left(n_t\left(\vec{c}_t,\vec{c}_{t-1}\right)\right) + \frac{\beta\theta_{t+1}}{\theta_t}V_{t+1}\left(\vec{c}_t\right)\right\} = \max_{R_{t+1}}\left\{U_t\right\}$$

where V_{t+1} is defined recursively, with V_{t+N} being the flexible price value function. We again have:

$$U_{c_t^j} = u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t}V_{c_t^j}$$

The envelope condition is:

$$V_{c_{t-1}^j} = \left(u_t\right)_{c_{t-1}^j} + \left(1-\delta^j\right)\frac{v_{n_t}}{f_{n_t}^j} = \left(u_t\right)_{c_{t-1}^j} + \left(1-\delta^j\right)p_t^j\lambda_t\left(1-\tau_t^j\right)$$

Thus we obtain:

$$\begin{aligned} U_{c_t^j} &= u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t}\left[\left(u_{t+1}\right)_{c_t^j} + \left(1-\delta^j\right)p_{t+1}^j\lambda_{t+1}\left(1-\tau_{t+1}^j\right)\right] \\ &= p_t^j\lambda_t\tau_t^j - \lambda_t\left(\frac{1-\delta^j}{R_{t+1}}\right)p_{t+1}^j\tau_{t+1}^j \end{aligned}$$

Now the optimality condition is just as before (after observing that $p^j_{t+1} = p^j_t$):

$$\sum_j \epsilon^j_R \gamma^j_t \left(\tau^j_t - \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau^j_{t+1} \right) = 0$$

which can also be written:

$$\sum_j \epsilon^j_R \gamma^j_t \tau^j_t = \sum_j \epsilon^j_R \gamma^j_t \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau^j_{t+1}$$

The expression for $N = 1$ is implied, after we note that in this case $\tau^j_{t+1} = 0$. □

Proof of Proposition 9. With commitment the central bank chooses the path of interest rates $\{R_k\}_{k=1}^N$ to maximize objective function:

$$\sum_{t=0}^{N-1} \beta^t \theta_t \left[u\left(\vec{c}_t, \vec{c}_{t-1}\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right) \right] + \beta^N \theta_N V_N\left(\vec{c}_{N-1}\right)$$

where $V_N\left(\vec{c}_{N-1}\right)$ is the flexible price value function entering period N. The optimal choice of R_k satisfies:

$$\sum_{t=0}^{N-1} \beta^t \theta_t \sum_j \left(u_{c^j_t} - \frac{v_{n_t}}{f^j_{n_t}} \right) \left(c^j_t \right)_{R_k} + \sum_{t=1}^{N} \beta^t \theta_t \sum_j \left[\left(\left(u_t\right)_{c^j_{t-1}} + \left(1-\delta^j\right) \frac{v_{n_t}}{f^j_{n_t}} \right) \left(c^j_{t-1} \right)_{R_k} \right] = 0$$

Adjusting the time indices yields:

$$\sum_{t=0}^{N-1} \beta^t \theta_t \sum_j \left\{ \left(u_{c^j_t} + \beta \frac{\theta_{t+1}}{\theta_t} \left(u_{t+1}\right)_{c^j_t} - \frac{v_{n_t}}{f^j_{n_t}} + \beta \frac{\theta_{t+1}}{\theta_t} \left(1-\delta^j\right) \frac{v_{n_{t+1}}}{f^j_{n_{t+1}}} \right) \left(c^j_t \right)_{R_k} \right\} = 0$$

Using $v_{n_t}/f^j_{n_t} = \lambda_t p^j_t \left(1-\tau^j_t\right)$ and $u_{c^j_t} + \beta \frac{\theta_{t+1}}{\theta_t} \left(u_{t+1}\right)_{c^j_t} = p^j_t \lambda_t - \frac{1-\delta^j}{R_{t+1}} p^j_{t+1} \lambda_t$, we can write this as:

$$\sum_{t=0}^{N-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma^j_t \epsilon^{y^j_t}_{R_k} \chi^j_t = 0$$

where

$$\chi^j_t = \tau^j_t - \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau^j_{t+1}$$

is the overhang-augmented labor wedge. □

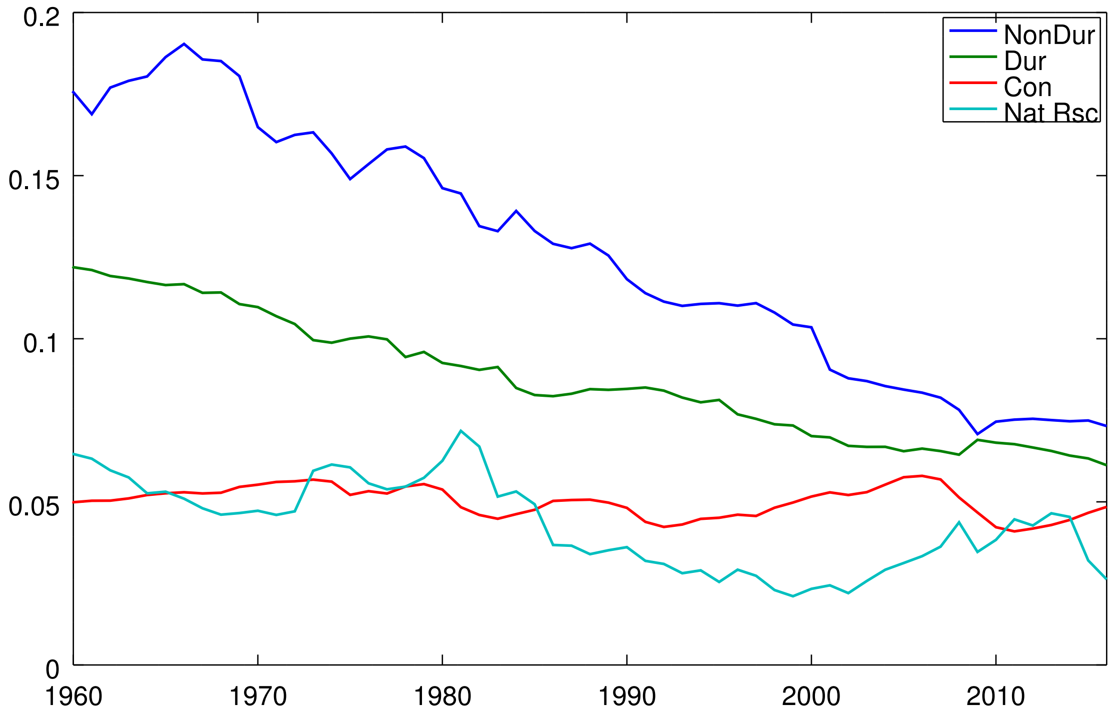

Figure 9: Sectoral GDP Shares

E Additional Figures

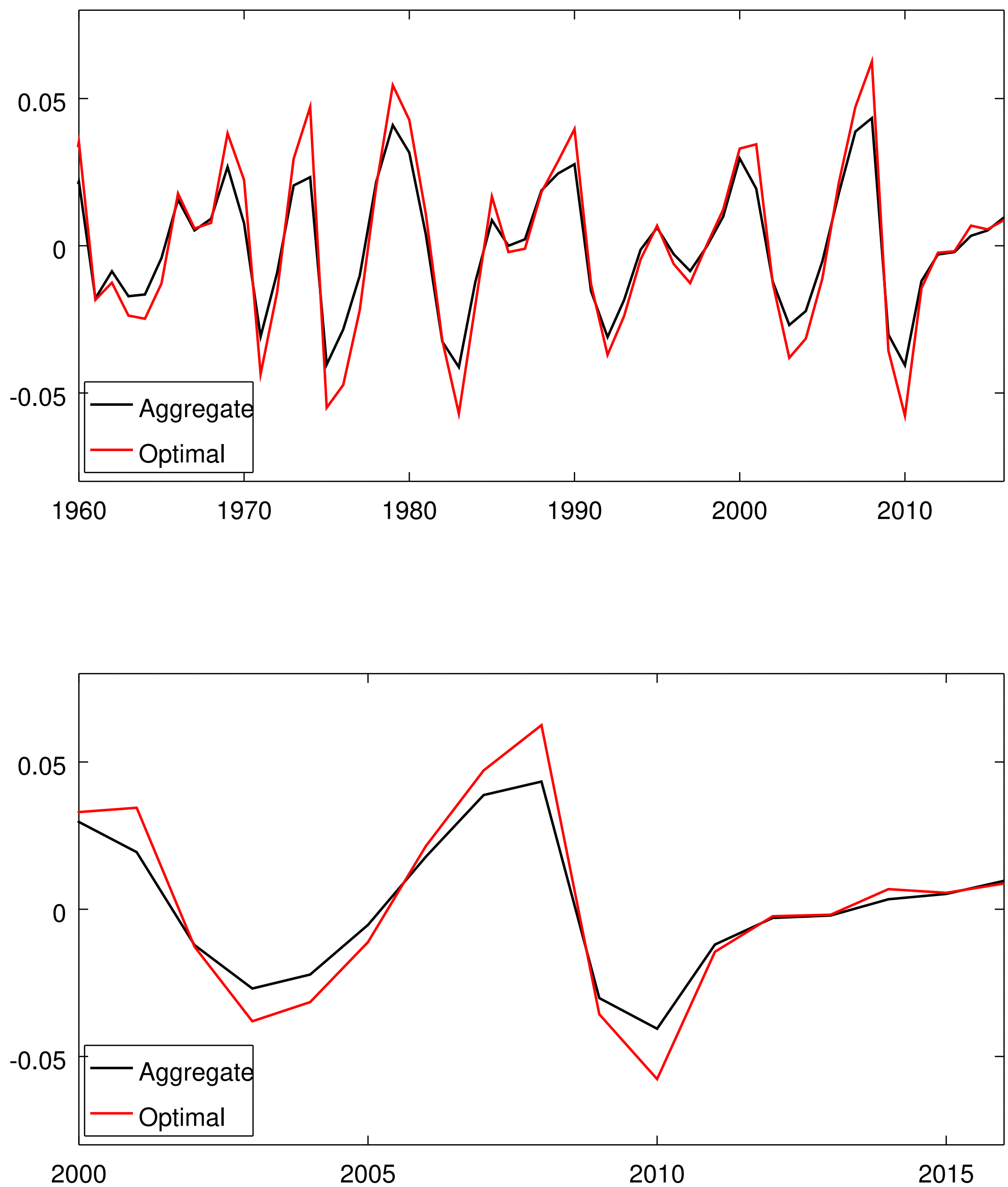

Figure 10: Aggregate vs. optimally-weighted inverse labor wedges, lower elasticity in construction

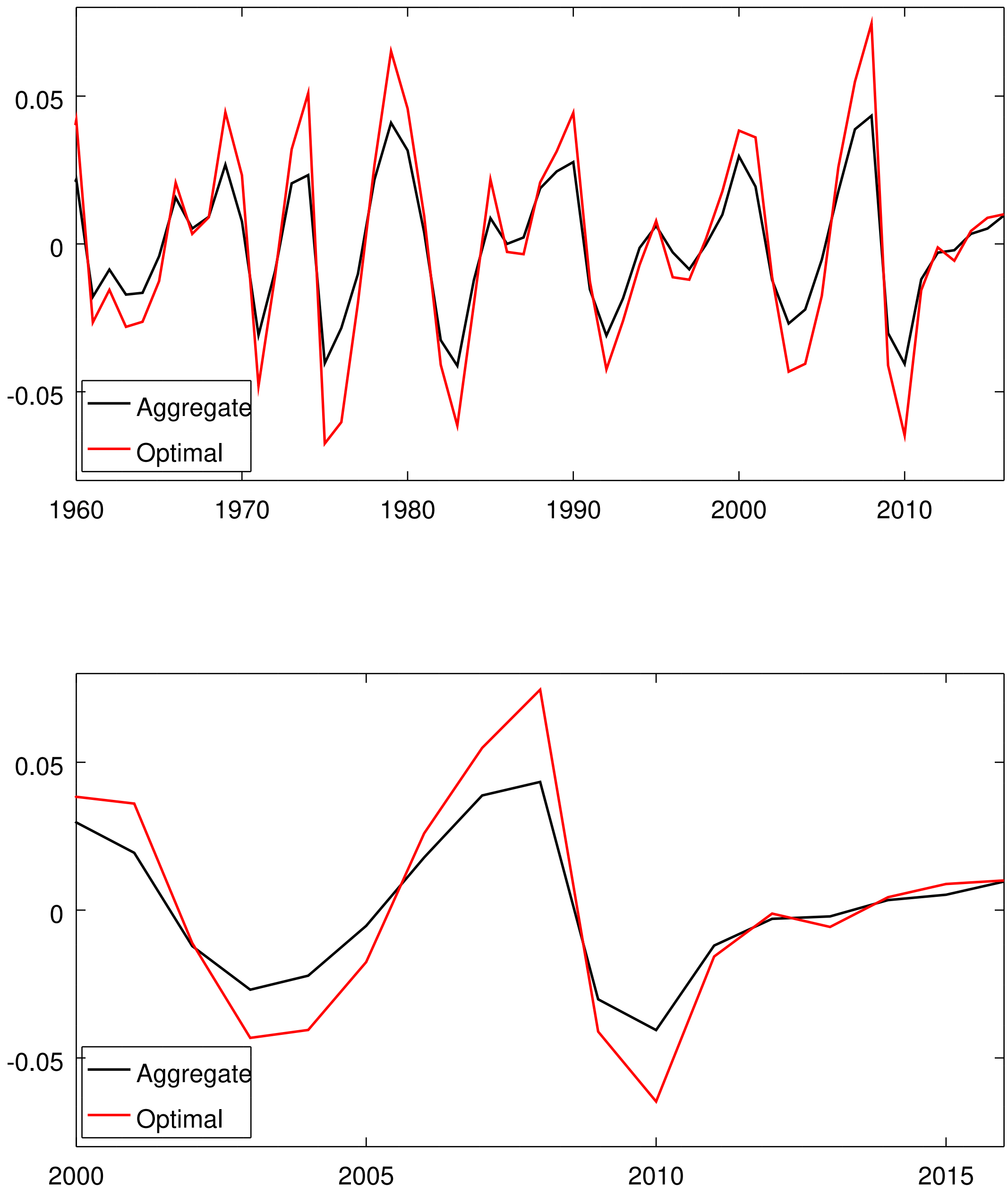

Figure 11: Aggregate vs. optimally-weighted inverse labor wedges, including natural resource sector