Sectoral Heterogeneity and Monetary Policy*

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Abstract

Since sectors differ in their sensitivity to interest rates, monetary policy produces inefficient sectoral fluctuations. In a model with sectoral heterogeneity, I show that policymakers should weight sectors proportionally to their interest elasticities, account for dynamic demand effects from durable goods, and systematically utilize forward guidance to reduce sectoral volatility. A calibrated model confirms these recommendations, and finds that neglecting sectoral volatility produces substantial welfare losses. The best-performing policy rule stabilizes a sectorally-weighted measure of inflation, plus lags of past nondurable inflation.

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1 Introduction

Economists have long understood that sectors of the economy differ in their sensitivity to interest rates, with durable consumer goods and investment being especially sensitive to interest rates.[1] Given this differential sensitivity, the conduct of monetary policy implies fluctuations in sectoral production which may be quite inefficient. Nevertheless, policymakers generally focus on aggregate factors when setting monetary policy, considering these sectoral dislocations second-order. In this paper, I argue that this is a mistake, and that monetary policy should take sectoral heterogeneity into account.

I develop the argument using both a simplified analytical model with multiple goods, and a calibrated quantitative two-sector model. I allow sectors to differ in the durability of produced goods, which produces differential interest sensitivity of demand across sectors. I find four main results. First, in the presence of sectoral heterogeneity and short-run imperfect substitution between sectors, monetary policy alone generically fails to achieve the first best. Second, it is optimal to stabilize a weighted average of sectoral labor wedges, where weights are proportional to sectoral interest elasticities. Third, policy should account for dynamic demand effects from durables, as greater durable production increases the stock of durables, decreasing future demand for new durables. Fourth, if central banks can commit to a future path of policy, they should systematically use forward guidance, since future interest rates produce smaller sectoral fluctuations than current interest rates. I prove these results in the analytical model, and then show that they hold in a calibrated quantitative model. The best-performing rule targets a measure of inflation that places a high weight on durable good inflation, and modifies the inflation target based on past nondurable inflation.

This paper consists of two main parts. In section 2, I work with an analytical model with many goods and a general utility function. Goods differ in their interest sensitivity both because of differential durability and differential interest-elasticity of substitution from the utility function. For tractability I assume a particular restrictive form of nominal rigidity: that prices are fixed for N periods and flexible thereafter. This allows the derivation of simple expressions for optimal monetary policy that yield intuitive interpretations. I find that optimal policy can be stated in terms of sectoral output shares, labor wedges, and interest elasticities. The baseline analysis proceeds under further simplifying assumptions, but I show in the appendix that many of these can be relaxed without qualitatively changing the results.

[1] For example, Boivin et al. (2010) find a limited response of consumption to a monetary policy shock, while there is a large effect on industrial production and housing starts, while Erceg and Levin (2006) estimate that "[A] monetary policy shock causes a decline in our broad measure of consumer durables spending that is over three times as large as for the other GDP components."

The first case I consider is that prices are fixed for one period only. I find that optimal policy satisfies what I call the static optimal policy rule:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

where γ^j is the GDP share of sector j, ϵ_R^j is the interest elasticity of demand, and τ^j is the labor wedge. If $\tau^j = 0$ for all sectors is feasible, then this is the solution; however, this will generally be impossible, since fixed prices implies fixed relative prices between sectors, whereas most shocks imply relative price movements. When stabilization of every sector is infeasible, the central bank must trade off stabilizing some sectors against others when setting interest rates. One might naively think that it is optimal to weight sectors by their GDP shares alone, as is done under inflation targeting.[2] However, this expression implies that sectors should be weighted by the interest elasticity, implying that more interest-sensitive sectors should receive a higher weight in setting monetary policy.

I next consider cases with $N > 1$, so that the central bank can set interest rates for multiple periods. Now the results depend on whether the central bank can commit to future policy. Without commitment, I derive an optimal policy rule that balances current labor wedges weighted by interest elasticities, and future labor wedges weighted by current interest elasticities and sectoral durability. This captures that production of durable goods in the current period increases the stock of durables in future periods, thereby reducing future demand in those sectors. I call this the durable overhang effect.[3]

I also consider cases with commitment. Commitment allows the central bank to set the future path of interest rates, rather than just the current rate. This allows the central bank to utilize forward guidance, i.e. to commit to changing future interest rates to affect demand today. If future interest rates had equivalent effects as current rates, then this would make no difference; however, if future rates systematically differ from current rates in their sectoral effects, forward guidance may be used to manage sectoral volatility. I calculate an optimal policy rule that differs from the no commitment case by the inclusion of a term proportional to the covariance of sectoral interest elasticities and the difference between interest elasticity with respect to current and future interest rates. In other words, if relative sectoral demand responds differentially to future and current interest rates, there are gains from using forward guidance. I show that under standard theory current interest rates produce greater sectoral volatility, since durable goods are more sensitive to current interest

[2] There is an analogy between labor wedges and inflation, since the labor wedge is proportional to the desired markup, and to the output gap in a linearized one-sector model.

[3] Rognlie et al. (2018), Beaudry et al. (2017), and McKay and Wieland (2019) discuss analogous effects, although their concern is with aggregate demand rather than sectoral factors.

rates than to future interest rates, whereas nondurable goods show no such difference. This implies that if a central bank is able to commit to a policy path, it should systematically use forward guidance in the conduct of monetary policy. This is equivalent to altering the inflation target based on past inflation.

In section 3, I replace the N-period fixed price assumption with Calvo pricing. This allows a more realistic treatment of inflation and the calibration of a quantitative example. I calibrate a two-sector model, with one sector corresponding to durable goods (e.g. consumer durables and residential investment) and the other sector corresponding to nondurable goods. I confirm that durable goods are more interest-sensitive than nondurable goods in the model, and that durable goods are more sensitive to current than to future interest rates. I also depart from the baseline analytical model by including uncertainty and durable adjustment costs. I include a number of shocks, including aggregate and sectoral TFP and cost-push shocks, and shocks to the discount rate, durable demand, and labor supply. I also examine the response of the model to monetary policy shocks, including both current and one-period ahead anticipated shocks, with the latter interpretable as a "forward guidance" shock.

I compare the performance of various monetary policy rules, as judged by the average expected welfare in the stationary distribution of the model. Since the model has two sectors that differ in sensitivity to shocks, under flexible prices the relative price of the two sectors is quite volatile. With nominal rigidities, the relative price cannot adjust as much, which generates costly sectoral fluctuations. Moreover, monetary policy cannot fully stabilize the economy, since the sectors differ in their sensitivity to the interest rate, and therefore the use of monetary policy itself produces sectoral volatility.

I compare several classes of policy rules. Among rules that make reference to current aggregate variables only, I find that inflation targeting performs quite well, while consideration of other variables (such as the output gap or wages) contributing little. When the policy rule can place different weights on inflation in each sector, it is optimal to weight the durable sector quite a bit higher than its GDP share, with the optimal weight increasing in the interest sensitivity of the sector (and therefore decreasing in the durable adjustment cost). There is little added benefit in targeting other aggregate variables when the central bank can target sectoral inflation, suggesting sectoral inflation is close to optimal for policies that set interest rates based on current variables only.

Finally, I explore whether it is possible to improve on sectoral inflation targeting by the systematic use of forward guidance. Since forward guidance involves committing to future interest rates based on current variables, it is equivalent to incorporating lagged variables into the current interest rate rule. I search for an optimal inflation target that is a weighted average of current and lagged sectoral inflation rates. I find that it is optimal to place significant weight on nondurable lagged inflation. For example, in the baseline calibration without adjustment costs, if nondurable inflation has averaged 1% over the past 3 quarters, it is optimal to lower the current inflation target by about 0.44%.

Literature. This paper is most closely related to the literature on optimal monetary in models with durable goods and substantial price rigidity across sectors. Erceg and Levin (2006) analyze a two-sector sticky-price model with durable goods that is similar to the one I consider in section 3. They show that their model fits a VAR quite well. They compare various simple policy rules, and find that inflation targeting performs poorly, while a policy that targets the aggregate output gap and a wage-price rule each perform well. In a recent working paper, Barsky et al. (2016) study monetary policy in a two-sector model with durable goods. They find that durable goods are highly interest-sensitive, and that the output gap is heavily dependent on durable inflation. They argue that monetary policy should place a heavy weight on durable inflation to stabilize the output gap, consistent with my findings. In another recent working paper, Basu et al. (2016) find similar result, although their model focuses on investment goods rather than consumer durables; nevertheless, the mechanism is essentially the same.

Relative to these papers, I focus on deriving optimal policy in a simpler model with a wider set of goods. This allows me to highlight the role of sectoral interest elasticity, and derive results concerning durable overhang, the role of commitment, and forward guidance that others have not found due to focusing only on comparing numerical methods and simple rules. Further, my result that the central bank should systematically utilize forward guidance to reduce sectoral volatility is entirely novel, as far as I can tell.

Beyond durable goods, there are a number of papers that consider optimal monetary policy with sectoral heterogeneity. Mankiw and Reis (2003) ask what price index a central bank should target. This question is quite similar to that considered in this paper, but the dimensions of sectoral heterogeneity considered are quite different. They allow sectors to differ by size, cyclical sensitivity, size sectoral shocks, and price flexibility, but not by interest elasticity (except implicitly through price flexibility). Moreover, they focus on the limiting case of infinite risk aversion, under which sectoral volatility vanishes in their welfare function, whereas I am chiefly concerned with this volatility.

In a similar vein, several papers have examined sectoral heterogeneity in the degree of price stickiness. Aoki (2001) study a two-sector model, with one flexible and one sticky sector, and argues that it is optimal to target inflation in the sticky price sector only. Carvalho et al. (2006) analyze sectoral heterogeneity in price stickiness from a positive perspective, and argue that it increases sluggishness of overall inflation. Bouakez et al. (2009) estimate a multisector DSGE model, and conclude that there is substantial sectoral heterogeneity in price stickiness.

Continuing the theme of heterogeneity in price stickiness, Barsky et al. (2007) document that a two-sector model with flexible durable prices implies negative sectoral comovement following a monetary policy shock. Since this is at odds with the data, they term this the comovement puzzle. Several papers have tried to resolve this puzzle by various methods, while others have simply assumed that the prices of durable goods are sticky as well. Notably, Bouakez et al. (2011) claim that including intersectoral demand linkages eliminates the comovement puzzle.

This paper is also related to the literature on monetary unions. The problem in a monetary union is that a single monetary policy is applied to heterogeneous regions, which is analogous to the problem of a single monetary policy with heterogeneous sectors. Benigno (2004) analyzes optimal monetary policy in a currency union, and finds that an inflation target that weights countries proportionally to their price rigidity is close to optimal.

Recently Bils et al. (2013) exploit the greater cyclical sensitivity of durable goods to test the "Keynesian labor demand hypothesis", i.e. that producers respond to low demand by cutting production rather than prices. They confirm that durable goods are highly cyclically sensitive in the data, and find countercyclical markups consistent with significant price stickiness in durable sectors. They conclude that this supports the Keynesian labor demand hypothesis.

2 Model without Inflation

I start by deriving the main results in an analytical model. The baseline model is highly general regarding the structure of production and household utility; however it abstracts from inflation and uncertainty, which allows the derivation of simple and interpretable expressions for optimal policy. The derivations in this section are extended to the case of uncertainty in appendix B, and section 3, develops a quantitative model with inflation.

2.1 Flexible Prices

The model is set in discrete time ($t = 0, 1, \dots$). There are N_j consumption goods, indexed by $j \in 1, \dots, N_j$. There is a unit measure of households, and a unit measure of firms in each sector. There is no capital, and the only asset is a nominal bond which is in zero net supply.[4]

Households. Households have lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \theta_t [u(\vec{c}_t) - v(n_t)] \tag{1}$$

where θ_t is a demand shock, $\vec{c}_t$ is a vector of consumption goods, and n_t is labor.[5] I assume that $u(\cdot)$ is strictly increasing and concave, and that $v(\cdot)$ is strictly increasing and convex.

Consumption goods may be durable. In particular, I assume that good j depreciates at the rate $\delta^j \in \{0, 1\}$. Goods in sector j have price p_t^j. Households supply labor n_t in return for wages w_t. They own firms and receive profits π_t^j from firms in sector j. There is a nominal bond that yields gross return R_{t+1} in period $t+1$. These assumptions yield household budget constraint:

$$a_{t+1} + \sum_j p_t^j \left[c_t^j - (1-\delta^j)c_{t-1}^j\right] = w_t n_t + \sum_j \pi_t^j + R_t a_t \tag{2}$$

Households maximize utility (1) subject to budget constraint (2). Household optimality conditions are:

$$w_t = \frac{v_{n_t}}{\lambda_t} \tag{3}$$

$$\lambda_t = \frac{\beta \theta_{t+1}}{\theta_t} R_{t+1} \lambda_{t+1} \tag{4}$$

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \left(\frac{1-\delta^j}{R_{t+1}}\right) p_{t+1}^j \tag{5}$$

where (3) and (4) are standard labor supply and Euler equations, and (5) is an asset-pricing condition for potentially durable consumption goods.

Interest-elasticity of demand. A central feature of the analysis is the differential sensitivity of sectoral demand to interest rates. Thus it is important to note that interest sensitivity is increasing in a sector's durability. To see this, suppose the utility function is separable. Then log-linearizing (4) and (5) and combining them yields:

$$\sigma^j \hat{c}_t^j = -\hat{\lambda}_{t+1} + \hat{\theta}_t - \hat{\theta}_{t+1} - \hat{p}_t^j - \hat{R}_{t+1} + \left(\frac{1-\delta^j}{r+\delta^j}\right)\left(\pi_{t+1}^j - \hat{R}_{t+1}\right)$$

[4] Since there is a representative household, the only role of this bond is to define the interest rate of the economy, which is the mechanism through which monetary policy is conducted.

[5] The model can be generalized to different types of labor.

where $r = R - 1$ is the steady state net interest rate.

Deviations in the interest rate $\hat{R}_{t+1}$ enter this expression in two places. The first is the standard aggregate demand effects operating through intertemporal substitution, as expressed in the Euler equation. This applies to all goods in proportion to their IES, regardless of durability. The second appearance captures additional demand effects on durable goods, which occur because durable goods are a means of saving as well as providing current consumption. A reduction in the interest rate R_{t+1} raises the present value of future income, which raises the value of durable good purchases to the household, and causes the household to demand more durable goods.

To focus on the effects of interest rates, suppose that demand shocks, changes in prices, and future income shocks are all zero. Then we have a partial equilibrium analog of the interest elasticity of demand for good j of:

$$-\frac{\hat{c}_t^j}{\hat{R}_{t+1}} = \left(\frac{R}{r + \delta^j}\right)\frac{1}{\sigma^j}$$

To see how much difference this can make to the interest elasticity, suppose that a good has a depreciation rate of 10% and that the interest rate is 5%, as in the calibration in Erceg and Levin (2006). Then the interest elasticity of demand for that good would be about 6 times greater than a non-durable good with the same IES.[6]

Firms. There is a unit interval of firms in each sector that produce using technology:

$$y_t^j = f_t^j(n_t^j) \tag{6}$$

where f_t^j satisfies the Inada conditions. Labor is perfectly substitutable across sectors. Thus firm profits are:

$$\pi_t^j = p_t^j y_t^j - w_t n_t^j \tag{7}$$

which implies optimal labor demand:

$$p_t^j = \frac{w_t}{f_{n_t}^j} \tag{8}$$

[6]This calculation neglects changes in relative prices that would occur in general equilibrium, and thus will be greater than the true elasticity of demand. On the other hand, assuming $\delta^j = 0.1$, a 1% change in demand for durables would imply a 10% change in the demand for *new* durables.

Equilibrium. The market clearing conditions are:

$$a_t = 0 \tag{9}$$

$$c_t^j = y_t^j + (1 - \delta^j)c_{t-1}^j \tag{10}$$

$$n_t = \sum_j n_t^j \tag{11}$$

Equilibrium is defined as follows:

Definition 1 (Flexible price equilibrium)**.** Given initial goods and demand shock $\{\bar{c}_{t-1}^j, \theta_t\}$, the equilibrium is a path of prices $\{p_t^j, w_t, R_{t+1}\}$ and quantities $\{c_t^j, n_t^j, n_t, y_t^j, \pi_t^j, a_t)\}$ that satisfy household conditions (3) - (5), firm conditions (6) - (8), and market clearing conditions (9) - (11).

The definition above does not pin down the level of nominal prices, since all equilibrium expressions will continue to hold if p_t^j and w_t are multiplied by a constant factor. To specify equilibrium, we choose good 1 to be the numeraire. Further, we will henceforth assume that good 1 is non-durable, so that $\delta^1 = 1$. This is a convenient assumption for expository purposes, but does not affect any results. Note that this implies that R is now the *real* interest rate in terms of good 1.

2.2 One-period fixed prices

To analyze the consequences of differential sectoral interest elasticity, we need some sort of sticky prices so that monetary policy has real effects. The simplest form of nominal rigidity is that the nominal prices of goods in all sectors are fixed for the current period, and flexible thereafter. Thus $p_t^j = \bar{p}^j$ for $t = 0$.

Following the normal convention in New Keynesian models, I assume that output is wholly demand determined. Thus all household optimality conditions continue to hold, but firm labor demand conditions (8) may not be satisfied:

$$w_t \neq p_t^j f_{n_t}^j$$

Since it is only the relative prices that matter for equilibrium, fixed prices in N sectors for one period determines $N - 1$ real variables. Since we have discarded N equilibrium conditions, we need one more expression to specify equilibrium. Thus I assume that the central bank chooses the real interest rate R_{t+1}.[7]

[7] I assume that the interest rate is chosen such that demand for new durable goods is always non-negative.

In periods $t \geq 1$, equilibrium is defined according to the flexible price equilibrium defined in section 2.1

Statement of Optimal Policy Problem. We next formulate the optimal policy problem. Consider the problem of the planner in period t, given fixed prices $\bar{p}_t^j$ and previous period goods $\vec{c}_{t-1}$. The next period value function $V_{t+1}(\vec{c}_t, \theta_{t+1})$ is known. The social planner then chooses R_{t+1} to maximize the welfare of the representative household.

First we define how demand for good j depends on interest rates. Since starting the next period the economy will be in the flexible price equilibrium, the functions $\vec{c}_{t+1}(\vec{c}_t, \theta_{t+1})$ and $\vec{p}_{t+1}(\vec{c}_t, \theta_{t+1})$ are known. Then we can combine (5) with fixed prices and the Euler equation to obtain:

$$u_{c_t^j} = \bar{p}_t^j u_{c_t^1} \left(1 - \left(\frac{1-\delta^j}{R_{t+1}}\right) \frac{p_{t+1}^j}{\bar{p}_t^j}\right) \tag{12}$$

where we make use of the fact that $\lambda_t = u_{c_t^1}$. This gives us N equations in N unknowns, with the equation for good 1 simply being the standard consumption Euler equation. Thus this expression defines $\vec{c}_t(R_{t+1})$.

Now we turn to expressing welfare as a function of $\vec{c}_t$. First we define aggregate labor supply as a function of $\{\vec{c}_t\}$ by inverting the production function in each sector:

$$n_t\left(\vec{c}_t\right) = \sum_j \left(f_t^j\right)^{-1} \left(c_t^j - \left(1-\delta^j\right) c_{t-1}^j\right) \tag{13}$$

Now we can express the optimal policy problem as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right) \right\}$$

where $V_{t+1}(\vec{c}_t)$ is the next period value function.[8]

This problem is well-defined since we are considering a case with flexible prices starting in the next period. Thus $V_{t+1}(\vec{c}_t)$ is just the lifetime utility of the representative household that enters the flexible price equilibrium with initial goods $(\vec{c}_t)$.

Optimal Policy. The following proposition describes the optimal monetary policy:

[8]This problem formally includes non-negativity constraints on the production of new goods. We do not need to consider these constraints explicitly because the unconstrained solutions will necessarily be interior since the production functions satisfy the Inada conditions.

Proposition 1 (Static Optimal Policy Rule). *The optimal choice of R_{t+1} satisfies:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

where $\epsilon_R^j = -\frac{dy_t^j}{dR_{t+1}}\frac{R_{t+1}}{y_t^j}$ *is the interest-elasticity of demand for new production in sector* j*,* $\gamma_t^j = \frac{p_t^j y_t^j}{\sum_j p_t^j y_t^j}$ *is the GDP share of sector* j*, and* $\tau_t^j = 1 - \frac{w_t}{p_t^j f_{n_{t+1}}^j}$ *is the labor wedge in sector* j*.*

Proof. See appendix D. □

That is, the optimal policy is to set the interest rate so that the weighted average labor wedge is zero, where sectoral labor wedges are weighted by the size of the sector and the interest-elasticity of that sector. I call this the *static policy rule*, since it reflects the static tradeoffs associated with setting interest rates.

The sectoral labor wedges captures the degree to which there is excess or deficient demand in a given sector. $\tau_t^j > 0$ corresponds to deficient demand, because it indicates that output is below the optimal level. τ_t^j can also be interpreted as the percent deviation from the optimal markup (here zero) in sector j. Thus $\tau_t^j > 0$ indicates an excess markup, suggesting that firms in sector j would like to lower prices, but are prevented from doing so by the nominal rigidity. This is consistent with the interpretation of deficient demand in the sector.

If we added a marginal amount of price flexibility, we would find that sectoral inflation was inversely proportional to the sectoral labor wedge, i.e. $\pi_t^j \propto -\tau_t^j$.[9] Thus an alternative interpretation of the policy rule is that it is optimal to target inflation, where the inflation target weights prices by the sectoral interest elasticity. Given that empirical estimates suggest substantially higher interest elasticity for durable goods and particularly housing, this suggests policymakers should place greater weight on inflation in these sectors when setting policy.

To understand the derivation of the static policy rule, simply note that $\gamma_t^j \tau_t^j$ is proportional to the marginal benefit of increasing demand for good j. Optimal monetary policy consists of setting the interest rate such that the net marginal benefit of cutting the rate equals the marginal cost of doing so. The marginal benefit is the marginal value of increasing demand in every sector that has $\tau_t^j > 0$, i.e. that is experiencing insufficient demand, times the sensitivity of demand to a marginal change in interest rates. The marginal cost is defined likewise for those sectors with $\tau_t^j < 0$.

[9]For example, we could suppose that a fraction ϕ of firms in each sector are allowed to reset their prices that period, and take the limit as $\phi \to 0$. Since prices are flexible the next period, the optimal pricing rule is to set $p_t^j = w_t / f_{n_t}^j = \left(\frac{1}{1-\tau_t^j}\right) p_t^j$, assuming an optimal markup of zero.

Welfare Properties of Optimal Policy. Now we discuss the welfare properties of optimal policy. The results are summarized in the following proposition:

Proposition 2 (Welfare Properties of Equilibrium under Optimal Policy)**.** *Suppose the economy is initially at a Pareto Optimum and experiences a shock. The resulting equilibrium under optimal policy is Pareto Optimal only when $\bar{p}^j = p_t^{j,flex}$ for all j. A sufficient condition for this is linear production in each sector together with no idiosyncratic sectoral supply shocks.*

Proof. See appendix D. □

Proposition 2 implies that monetary policy alone is able to achieve the first best only in very particular circumstances. Since most economic shocks will imply some changes in relative prices in the flexible price equilibrium, it follows that the constraint of fixed relative prices will bind. The exception is the case of linear production in each sector, which implies that relative prices do not change in response to shocks, with the exception of sectoral productivity shocks. This is an extreme assumption, since it implies perfect substitution of productive factors between sectors.

Now that we have established the second-best nature of pure monetary stabilization, we turn to characterizing optimal monetary policy.

2.2.1 Tradeoff between aggregate and sectoral stabilization.

To characterize optimal policy, it is useful to express the policy rule in the following form:

Proposition 3 (Static Optimal Aggregate Sectoral Tradeoff)**.** *The static policy rule can be written as:*

$$\tau^y + \left(\epsilon_R^y\right)^{-1} \sum_j \gamma^j \left(\epsilon_R^j - \epsilon_R^y\right)\left(\tau^j - \tau^y\right) = 0$$

where:

$$\epsilon_R^y = \sum_j \gamma^j \epsilon_R^j$$
$$\tau^y = \sum_j \gamma^j \tau^j$$

Proof. See appendix D. □

This representation of the static optimal policy rule highlights the tradeoff between aggregate and sectoral stabilization. τ_t^y is the aggregate labor wedge, and ϵ_R^y is the aggregate interest-elasticity of demand. Since γ^j are sectoral weights, the second term is the covariance between sectoral interest-elasticity and labor wedges.

This expression indicates that when relatively interest-sensitive sectors are experiencing relatively high demand, it is optimal to tolerate deficient aggregate demand, rather than excessively stimulate these sectors by cutting interest rates further. It further indicates that aggregate stabilization is optimal when either all sectors are equally interest-sensitive, or all sectors have zero labor wedges when the aggregate labor wedge is zero, or when there is no correlation between interest elasticity and relative labor wedges. Since none of these cases is likely to hold in practice, we can say that the general result is that it is rarely optimal to target a zero aggregate labor wedge while neglecting sectoral considerations.

2.3 N-period Fixed Prices without Commitment

Now we extend the model to the case where prices may be sticky for multiple periods. Suppose that prices of goods in all sectors are fixed for N periods, and flexible thereafter. Thus $p_t^j = \bar{p}^j$ for $t < N$. As in the previous section, output is demand determined, household optimality conditions hold, and firm labor demand conditions may fail to hold for $t \leq N$. The central bank now chooses interest rates in every period with fixed prices.

Since the central bank is now choosing interest rates for several periods, we have to consider whether the central bank is able to commit to a future path of interest rates. I assume in this section that the central bank cannot so commit, and is limited to only choosing the current interest rate. I consider commitment in the next section.

Statement of Optimal Policy Problem. We formulate the optimal policy problem analogously to the previous section. Consider the problem of the planner in period t, given fixed prices $\bar{p}_t^j$ and previous period goods $\vec{c}_{t-1}$. The next period value function $V_{t+1}(\vec{c}_t, \theta_{t+1})$ is known. The social planner then chooses R_{t+1} to maximize the welfare of the representative household. Equations (12) and (13) continue to hold.

Now we can express the optimal policy problem without commitment recursively as:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{ u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \frac{\beta\theta_{t+1}}{\theta_t} V_{t+1}\left(\vec{c}_t\left(R\right)\right) \right\}$$

where $V_{t+1}(\vec{c}_t)$ is the next period value function.

For $t = N$, this problem is just the same as the one-period fixed price problem analyzed in the previous section. For $t < N$, the problem is different, and the next period value function V_{t+1} is defined recursively. This problem is well-defined since V_N is well-defined, and we can then define each previous period's problem by backward induction, where $V_{t+1}(\vec{c}_t)$ represents the value function under future optimal policy. Since the social planner lacks commitment, it cannot credibly promise to alter future policy, and thus takes $V_{t+1}(\vec{c}_t)$ as given in its choice of R_{t+1}.

Optimal Policy. The following proposition describes the optimal monetary policy:

Proposition 4 (Optimal Policy Without Commitment)**.** *The optimal choice of R_{t+1} satisfies:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

where $\epsilon_R^j = -\frac{dy_t^j}{dR_{t+1}} \frac{R_{t+1}}{y_t^j}$ is the interest-elasticity of demand for new production in sector j, $\gamma_t^j = \frac{p_t^j y_t^j}{\sum_j p_t^j y_t^j}$ is the GDP share of sector j, and $\tau_t^j = 1 - \frac{w_t}{p_t^j f_{n_{t+1}}^j}$ is the labor wedge in sector j.

Proof. See appendix D. □

The left-hand side of the optimal policy expression is the marginal value of cutting interest rates due to the effects on demand in the current period. When the right-hand side is zero, this is just the same as the static policy rule that prevails under one-period fixed prices.

The right-hand side is the dynamic marginal cost of cutting interest rates, deriving from entering the next period with a higher stock of durable goods. Holding desired consumption fixed, a higher initial stock of durable goods implies lower demand for new output. If there is overall deficient demand in durable sectors in the next period, as indicated by $\tau_{t+1}^j > 0$, then cutting interest rates in the current period carries a cost since it reduces effective future demand in these sectors. Thus it will be optimal to not cut interest rates as much as static considerations alone would imply, allowing a current weighted output gap $\sum_j \epsilon_R^j \gamma_t^j \tau_t^j > 0$. I call this the *durable overhang effect*.

2.4 N-period fixed prices with commitment

The analysis of optimal policy in section 2.3 assumed no commitment by the central bank. This turns out to matter because demand responds differentially to current and future interest rates. Differential sectoral sensitivity is greater with respect to current interest rates than with respect to future interest rates. Thus to minimize sectoral distortions, it is optimal to use forward guidance to minimize fluctuations in current interest rates.

First we formulate the problem. The economy enters period 0 with previous consumption vector $\vec{c}_{t-1}$. The monetary authority then chooses interest rates for the next N periods

$\{R_s\}_{s=1}^N$. We may state the problem as:

$$V_0(\vec{c}_{-1}) = \max_{\{R_s\}_{s=1}^N} \left\{ U_0(\vec{c}_1, \vec{c}_0) + \sum_{t=1}^{N-1} \frac{\beta^s \theta_t}{\theta_0} U_t(\vec{c}_t, \vec{c}_{t-1}) + \frac{\beta^N \theta_N}{\theta_0} V_N(\vec{c}_{N-1}) \right\}$$

where $U_t(\vec{c}_t, \vec{c}_{t-1}) = u(\vec{c}_t) - v(n_t(\vec{c}_t, \vec{c}_{t-1}))$ is the period utility function, and V_N is the flexible price value function. $\vec{c}_t$ are period demand functions.

Optimal Policy. We now turn to solving the optimal policy problem. First I show that demand in period t depends only on future interest rates, not on past interest rates.

Lemma 1 (Dependence on future interest rates only)**.** *Period t demand is a function only of fixed relative prices and the future path of interest rates, $\vec{c}_t\left(\vec{p}, \{R_s\}_{s \geq t+1}\right)$. In particular, c_t^j does not depend on past interest rates, so $\epsilon_{R_k}^{y_t^j} = 0$ for $k \leq t$.*

Proof. See appendix D. □

This lemma might be surprising, as one might think that cutting current interest rates affects future demand through changing the stock of future durables. That is, a lower interest rate in period t implies a greater stock of durables in period $t+1$. But with fixed prices and a given interest rate, there is simply no mechanism through which this greater stock of durables increases demand. It simply reduces effective demand for new durables, with the loss in income to households from lower production exactly offsetting the higher income from a greater stock of durables.

With Lemma 1 in hand, we use similar techniques to the last two sections to derive an expression for optimal policy:

Proposition 5 (Optimal policy with commitment)**.** *Optimal policy satisfies:*

$$\sum_{t=0}^{k-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_k}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \tau_{t+1}^j \left(\frac{1 - \delta^j}{R_{t+1}} \right)$$

is the durable overhang-augmented sectoral labor wedge.

Proof. See appendix D. □

Simplification of Optimal Policy Expression. The expression derived in Proposition 5 is difficult to interpret by itself. Start with the choice of R_1, i.e. the interest rate in the initial period. The expression is just the same as the without commitment:

$$\sum_j \gamma_0^j \epsilon_{R_1}^{y_0^j} \chi_0^j = 0$$

Intuitively, since the choice of R_1 only affects demand in period 0, the presence or absence of commitment does not matter.[10]

What about for the choice of R_k with $k > 1$? Here we can simplify by taking the difference between the optimality expressions for R_k and R_{k-1}. The key is that the elasticities of demand with respect to interest rates s periods ahead are the same for all $s > 1$, as proved in the following Lemma.

Lemma 2 (Symmetric effects of forward guidance)**.** $\epsilon_{R_k}^{y_t^j} = \epsilon_{R_\ell}^{y_t^j}$ *for* $k, \ell \in [t+2, N]$.

Proof. See appendix D. □

With Lemma 2 in hand, we can derive a simplified form of the optimality condition.

Proposition 6 (Simplified Optimal Policy Expression)**.** *For* $k \in [2, N]$*, combining the optimal choice of* $\{R_k, R_{k-1}\}$ *yields optimality condition:*

$$\sum_j \gamma_{k-1}^j \epsilon_{R_k}^{y_{k-1}^j} \chi_{k-1}^j = \frac{R_{k-1} y_{k-2}}{y_{k-1}} \sum_j \gamma_{k-2}^j \left(\epsilon_{R_{k-1}}^{y_{k-2}^j} - \epsilon_{R_k}^{y_{k-2}^j} \right) \chi_{k-2}^j \tag{14}$$

Proof. See appendix D. □

The key term here is $\epsilon_{R_{k-1}}^{y_{k-2}^j} - \epsilon_{R_k}^{y_{k-2}^j}$. For example, a positive term indicates that sector j is more sensitive to current interest rates than to future interest rates. Thus the righthand side of (14) is the covariance between the sectoral dynamic labor wedge and the sectoral relative sensitivity to current interest rates. When this term is 0, the optimality expression is just as in the no-commitment case. Thus the deviation from the no-commitment benchmark is proportional to the covariance between dynamic sectoral output gaps and differential sectoral interest elasticities with respect to current and future interest rates.

This expression governs the optimal use of forward guidance. The lefthand side is negative when forward guidance is employed, because this indicates that R_k is set lower than implied by the no-commitment policy rule in period $k-1$. This occurs when the righthand side is also

[10] Of course, this does not imply that the choice of R_1 will be the *same* as it would be without commitment. Since the choice of future interest rates will differ, current and future labor wedges will differ as well. It is merely the optimal policy expression that is unchanged.

negative — i.e., when past labor wedges are negatively correlated with relative sensitivity to current interest rates.

Sectoral variation in sensitivity to current and future interest rate. The optimal policy expression above suggests that it is optimal to routinely use forward guidance in the conduct of monetary policy as long as there is some systematic relationship between sectoral output gaps and sectoral relative sensitivity to current and future interest rates. In fact this is the case — generally speaking, durable goods are less sensitive to future interest rates than they are to current interest rates, whereas nondurable goods are equally sensitive to current and future interest rates.

The intuition for this is straightforward. Consider the pricing equation for good j:

$$u_{c_t^j} = \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j u_{c_t^1}$$

The current interest rate R_{t+1} appears directly in the expression, with its influence increasing in durability. By contrast, future interest rates enter through the Euler equation, i.e. through $u_{c_t^1}$. That is, cutting future interest rates lowers $u_{c_{t+1}^1}$, which in turn lowers $u_{c_t^1}$. This channel works independently of durability. We can prove this in the case of separable utility:

Proposition 7 (Durability and Forward Guidance)**.** *Suppose that $u(\vec{c}_t)$ is separable in c_t^j. Then we have:*

$$\epsilon_{R_{t+1}}^{y_t^j} - \epsilon_{R_{t+2}}^{y_t^j} = \left(\frac{1-\delta^j}{R_{t+1}}\right) \epsilon_{R_{t+1}}^{y_t^j}$$

and optimal policy satisfies:

$$\sum_j \gamma_{k-1}^j \epsilon_{R_k}^{y_{k-1}^j} \chi_{k-1}^j = \frac{R_{k-1} y_{k-2}}{y_{k-1}} \left[\sum_j \gamma_{k-2}^j \left(\frac{1-\delta^j}{R_{k-1}}\right) \epsilon_{R_{k-1}}^{y_{k-2}^j} \chi_{k-2}^j\right]$$

Proof. See appendix D. □

Proposition 7 suggests that the durable goods are relatively more sensitive to *current* interest rates than to *future* interest rates. This suggests that when durable goods are relatively overstimulated, the central bank should promise to cut future interest rates (instead of current rates), and when durable goods suffer relatively low demand, the central bank should promise to raise future interest rates and cut current interest rates further instead.

3 Quantitative Model with Inflation

The previous section derived general results for optimal policy under a particularly restrictive price-setting assumption: that prices were fixed for several periods, and then flexible thereafter. This assumption enabled the derivation of simple and clear expressions for optimal policy with straightforward interpretations. However, abstracting from inflation has two shortcomings. First, it is unclear whether the general qualitative results derived in the previous section are robust to the inclusion of inflation, and particularly costs of inflation. Second, it is difficult to assess the quantitative importance of sectoral considerations in such a simplified model. To address these concerns, this section analyzes a calibrated two-sector model with Calvo pricing. The results align with the conclusions of the previous section, and support the quantitative importance of sectoral considerations in setting monetary policy.

3.1 Model

I depart from the model in section 2 by allowing uncertainty. There are two goods: a nondurable consumption good c, and a durable consumption good d. The relative price between the sectors by $q_t = p_t^d/p_t^c$, and all others prices are expressed in terms of the non-durable good.

Households. Household utility has functional form:

$$\mathcal{U} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \theta_t \left\{ \frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \chi_t \frac{d_t^{1-\sigma_d}}{1-\sigma_d} - \psi_t \frac{n_t^{1+\zeta}}{1+\zeta} \right\} \right]$$

Note that utility is separable in both consumption goods, with the goods potentially varying in their intertemporal elasticity of substitution. $(\theta_t, \chi_t, \psi_t)$ are time-varying and stochastic, with the random component corresponding to shocks to the household discount factor, durable demand, and labor supply respectively.

There is a quadratic adjustment cost in durable good accumulation, such that increasing durable consumption requires the purchase of additional durable goods that are lost. The household budget constraint is therefore:

$$c_t + q_t d_t + a_{t+1} = \frac{R_{t-1}}{\Pi_t^c} a_t + w_t n_t + \pi_t + (1-\delta)\, q_t d_{t-1} - \frac{\varphi}{2} q_t \left(d_t - d_{t-1}\right)^2$$

where R_t is now the *nominal* gross interest rate on bonds (which are in zero net supply). The budget constraint is expressed with the non-durable good c_t as the numeraire. $\Pi_t^c = p_t^c/p_{t-1}^c$ is the gross inflation rate of the non-durable good.

Under these assumption household optimality conditions are:

$$1 = \mathbb{E}_t \left[\frac{\beta \theta_{t+1}}{\theta_t} \frac{R_t}{\Pi^c_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma_c} \right] \tag{15}$$

$$w_t = \psi_t n_t^{\zeta} c_t^{\sigma_c} \tag{16}$$

$$q_t = \chi_t \frac{d_t^{-\sigma_d}}{c_t^{-\sigma_c}} - \varphi q_t \left(d_t - d_{t-1} \right) + \beta \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma_c} q_{t+1} \left\{ 1 - \delta + \varphi \left(d_{t+1} - d_t \right) \right\} \right] \tag{17}$$

Supply Side. Suppose that in sector j there is a unit interval of intermediate good firms indexed by i.[11] The output of intermediate good firms is combined by a Dixit-Stiglitz aggregator firm to produce final goods:

$$y_t^j = \left(\int_i \left(y_{it}^j \right)^{\frac{\epsilon - 1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon - 1}}$$

Intermediate goods are produced by firms with Cobb-Douglas production functions and fixed capital normalized to 1. Thus the production function of firm i in sector j is:

$$y_{it}^j = z_t^j \left(n_{it}^j \right)^{1 - \alpha_j}$$

where z_t^j is sectoral productivity, and α_j is sectoral capital share.

Intermediate good firms are assumed to be subject to a Calvo pricing nominal rigidity. Every period, a random fraction $1 - \phi_j$ of firms in sector j are able to adjust their prices, while the remainder cannot. All firms must hire sufficient labor to meet the demand for the output they face at their posted price. To remove the inefficiency due to the market power of intermediate goods, I assume that intermediate good firms are provided a production subsidy $1 + \tau = \frac{\epsilon}{\epsilon - 1}$, proceeds from which are rebated in lump sum payments equally to all firms. Firms discount real profits at time t by $Q_t = \beta^t \theta_t c_t^{-\sigma_c}$. Under these assumptions, firms in sector j that can adjust their price will choose:

$$\left(p_t^{j*} \right)^{1 + \frac{\epsilon \alpha_j}{1 - \alpha_j}} = \frac{1}{1 - \alpha_j} \frac{X_t^j}{Z_t^j}$$

[11]Since this section is fairly standard, I omit many details of the derivation. They can be found in appendix C.

where X_t^j and Z_t^j are defined recursively as

$$X_t^j = \left(\Pi_t^j\right)^{\frac{\epsilon}{1-\alpha_j}} \left\{\mu_t^j w_t \left(y_t^j/z_t^j\right)^{\frac{1}{1-\alpha_j}} + \mathbb{E}_t\left[\frac{\beta\phi_j\theta_{t+1}}{\theta_t}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma_c} X_{t+1}^j\right]\right\}$$
$$Z_t^j = \left(\Pi_t^j\right)^{\epsilon-1} \left\{y_t^j + \mathbb{E}_t\left[\frac{\beta\phi_j\theta_{t+1}}{\theta_t}\left(\frac{c_t}{c_{t+1}}\right)^{\sigma_c} Z_{t+1}^j\right]\right\}$$

where μ_t^j is a sectoral markup shock.

Inflation. Suppose that all firms follow this pricing rule. At every point in time, we can distinguish between the aggregate price p_t^j, and the price of adjusting firms p_t^{j*}. The aggregate price level evolves according to:

$$\left(p_t^j\right)^{1-\epsilon} = \phi_j \left(p_{t-1}^j\right)^{1-\epsilon} + (1-\phi_j)\left(p_t^{j*}\right)^{1-\epsilon}$$

We can write this is inflation terms as:

$$\left(\Pi_t^j\right)^{1-\epsilon} = 1 + (1-\phi_j)\left[\left(\Pi_t^{j*}\right)^{1-\epsilon} - 1\right]$$

where $\Pi_t^j = p_t^j/p_{t-1}^j$ and $\Pi_t^{j*} = p_t^{j*}/p_{t-1}^j$.

Price Dispersion. Given the convexity of the sectoral CES aggregator, price dispersion within a sector lowers output for given inputs. In particular, sectoral output y_t^j is related to total sectoral labor n_t^j by:

$$y_t^j = z_t^j \left(\frac{n_t^j}{\Delta_t^j}\right)^{1-\alpha_j}$$

where Δ_t^j is sectoral price dispersion, which satisfies:

$$\Delta_t^j = \int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha_j}}$$

Price dispersion evolves over time according to:

$$\Delta_t^j = \left(\Pi_t^j\right)^{\frac{\epsilon}{1-\alpha_j}} \left[\phi_j \left(\Delta_{t-1}^j\right) + (1-\phi_j)\left(\Pi_t^{j*}\right)^{-\frac{\epsilon}{1-\alpha_j}}\right]$$

Market Clearing. Market-clearing conditions of the model are as follows:

$$
\begin{aligned}
y_t^c &= c_t \\
y_t^d &= d_t - (1-\delta)\, d_{t-1} + \frac{\varphi}{2}\left(d_t - d_{t-1}\right)^2 \\
a_t &= 0 \\
n_t &= n_t^c + n_t^d
\end{aligned}
$$

Shocks. Altogether the model above has seven shocks: $\left\{\theta_t, \psi_t, \chi_t, z_t^j, \mu_t^j\right\}$. The sectoral shocks z_t^j and μ_t^j are allowed to contain both an aggregate and sector-specific component, which combine multiplicatively, i.e. $z_t^j = z_t \tilde{z}_t^j$ and $\mu_t^j = \mu_t \tilde{\mu}_t^j$. All shocks are assumed to follow a log-normal mean-zero AR(1) process:

$$
x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_t^x
$$

for shock x.

3.2 Calibration

The calibration used in the baseline example is shown in Table 1. The model is quarterly. $\sigma_j = 1$, $\zeta = 1$ and $\alpha_j = 1/3$ are conventional choices. β is set so that the steady state annual interest rate is 3%. $\delta = 0.025$ gives 10% annual depreciation, which is the average depreciation rate of durable consumer goods (including housing). $\bar{\chi}$ (i.e. the mean value of χ_t) is set to target a durable GDP share of 1/8. The choice of target interest rate, depreciation rate, and durable share are as in Erceg and Levin (2006).

Note that this durable share is quite small relative to a reasonable interpretation of the data. For example, in 2018 total consumption expenditures less durable goods were $12.49 trillion, whereas expenditures on durable goods plus private fixed investment (residential plus nonresidential) were $5.05 trillion. This implies a durable share of 28.8%. Nevertheless, I stick with the lower value both because it is more conservative (since a lower durable share implies a lower cost of sectoral volatility), and for greater comparability with past work.

$\bar{\psi}$ is chosen so that steady state labor is 0.8, consistent with the average prime-age employment rate. $\epsilon = 6$ and $\phi_j = 2/3$ are as recommended in Gali's textbook; the latter

σ_j	α_j	ζ	δ	β	ϕ_j	ϵ	$\bar{\chi}$	$\bar{\psi}$	$\bar{z}^j$	φ	σ_x^2	ρ_x
1	$\frac{1}{3}$	1	0.025	0.9926	$\frac{2}{3}$	6	0.18	1.19	1	0	$1e-5$	0.95 or 0.8

Table 1: Calibration of Quantitative Example

corresponds to prices adjusting every 3 quarters on average, consistent with evidence from micro data. $\bar{z}^j = 1$ is a normalizations. I consider several levels of adjustment costs, corresponding to different levels of φ, but the baseline version has no adjustment costs.

The stochastic processes are all assumed to have the same variance, corresponding to a variance of $\sigma_x^2 = 1e-5$. The persistence of shocks is $\rho_x = 0.95$ for firm shocks $x \in \left\{z_t, \tilde{z}_t^j, \mu_t, \tilde{\mu}_t^j\right\}$, and $\rho_x = 0.8$ for household shocks $x \in \{\theta_t, \psi_t, \chi_t\}$.

Section 3.6 explores the consequences of varying these parameters, and the individual effects of particular shocks on the results.

Note that this calibration implies that durable goods are just as sticky as nondurable goods. This is a controversial question. Erceg and Levin (2006) also assume identical price stickiness; by contrast, Barsky et al. (2007) argue that housing prices are quite flexible. Álvarez et al. (2006) examine micro price data in the Euro area and conclude that prices of durable and capital goods are stickier than non-durable goods. Cantelmo and Melina (2018) find using a SVAR that relative durable good prices respond little to a monetary policy shock, implying a similar degree of price stickiness between sectors.

3.3 Model Dynamics under Taylor Rule

The description of the model above did not specify the policy rule. In the following sections, we will compare a number of different rules. We begin with Taylor rules of the form:

$$R_t = R^* \left[\Pi_{t+1}^c\right] (\Pi_t)^{\omega_\pi} \left(y_t / y_t^f\right)^{\omega_y} + \varepsilon_t + \nu_{t-1} \tag{18}$$

where R^* is the steady state real interest rate, where y_t^f is the flexible price levels of output, and where ε is a monetary policy shock. Thus the interest rate reacts to current inflation and to the output gap.[12]

I use weights $\omega_\pi = 2$ and $\omega_y = 0.1$ as my baseline Taylor rule. This is very close to the weights estimated in Smets and Wouters (2007), although their rule includes an interest rate smoothing dynamic that I do not use. This rule implies a somewhat stronger reaction to current inflation than the 1.5 value found by Taylor (1993) and used by Christiano et al. (2005). In the current model, a stronger reaction to inflation results in higher welfare overall, and thus using $\omega_\pi = 2$ is more conservative in the sense that it casts the Taylor rule in a better light.

To study the dynamics of the model, I simulate the model for 10,000 periods using

[12] The aggregate inflation rate is calculated as the average of sectoral inflation rates weighted by steady state GDP shares, i.e. $\Pi_t = (\Pi_t^c)^{\gamma_c} \left(\Pi_t^d\right)^{1-\gamma_c}$, which in the baseline calibration is $\gamma_c = 7/8$. GDP is calculated using the nondurable good as a numeraire, i.e. $y_t = y_t^c + q_t y_t^d$.

	Data	$\varphi = 0$ sticky	$\varphi = 0$ flex	$\varphi = 0.3$ sticky	$\varphi = 0.3$ flex	$\varphi = 0.6$ sticky	$\varphi = 0.6$ flex
y	1.71	1.78	1.54	1.65	1.53	1.61	1.52
c	0.85	1.47	1.43	1.47	1.44	1.47	1.44
y^d	4.72	6.42	2.66	3.90	2.50	3.14	2.35
n	1.79	1.15	0.33	0.86	0.32	0.79	0.31
w	0.68	2.11	1.48	1.95	1.48	1.92	1.48
q	1.61	1.14	1.38	1.16	1.37	1.17	1.37
Π	2.10	0.46		0.45		0.45	
Π^c	1.63	0.44		0.45		0.46	
Π^d	3.96	1.00		0.77		0.71	

Table 2: Standard Deviation of Log Variables along Simulated Path. Source: King and Rebelo (1999) and author's calculations

a second-order approximation. Table 2 gives the moments of the baseline sticky price and flexible price models for three different values of the adjustment cost parameter φ. All models are computed using the same initial random seed, with flexible price variables calculated in response to precisely the same shocks as the sticky prices models. Thus all reported differences reflect differential responses of the models to the same set of shocks. The reported numbers are the standard deviation of log deviation from trend of each variable, multiplied by 100 so the values can be viewed as percents.

The first column shows comparable moments from the data. The standard deviation of labor and wage are as reported by King and Rebelo (1999). Production and inflation numbers were calculated directly from the data. Output of nondurable goods includes two components of consumption: services and nondurables. Durable production includes durable consumption plus private fixed investment (residential and non-residential). Each component was deflated by its implicit price deflator and combined, which implied price indices for durable and nondurable goods.[13] The resulting data series were then hp-filtered after taking logs. Total output is here the sum of the two components, rather than GDP, and thus corresponds closely consumption plus investment. Likewise inflation is the GDP-share weighted average of the two implicit price deflators, and therefore does not correspond exactly to other standard measures of inflation.

The volatility of total output in the model is quite similar to that found in the data. The volatility of nondurable production is a bit too high. The volatility of durable production is too high in the absence of adjustment costs, and fairly similar for moderate adjustment costs ($\varphi = 0.3$). The volatility of wages is too high, and of labor too low, probably due

[13]Note, as discussed in the previous section, the durable share according to this definition is about 0.315, whereas the model is calibrated to a durable share of 0.125, following Erceg and Levin (2006).

to the assumption of flexible wages. The volatility of the relative price of durables is a bit lower than in the data. Perhaps the greatest discrepancy among non-labor market variables is that the volatility of inflation is quite a bit lower than in the data, though the relative magnitudes between sectors is similar.

Figure 1 shows the response of key variables to several real shocks in the flexible price and sticky price models. The first column shows the response to a shock to θ. This is equivalent to a short-term fall in the discount factor, which increases desired saving. In the flexible price model, this results in a fall in the real interest rate, and a shift in production towards durable goods, which are useful for transferring resources into the future. This increase in durable production drives up the relative cost of producing durable goods, which raises the relative price of durables, substantially mitigating the relative shift in demand. In the sticky price model, the relative price of durables cannot fully adjust, and therefore a given reduction in the real interest rate leads to a much greater increase in durable production, at substantial efficiency losses.

The second and third columns of figure 1 show the economy's response to aggregate and non-durable sector-specific TFP shocks respectively. In both cases there is a temporary increase in production due to higher productivity, and also an increase in desired saving, as households try to smooth their consumption by saving some of their temporarily increased income. As in the case of the demand shock, this increased desire to save causes both a decline in real interest rates and an increase in relative demand for durable goods, leading to an increase in relative durable prices. However, when prices are sticky this adjustment cannot happen easily, and so there is excess production in the durable sector and underproduction of nondurables, leading to efficiency losses. This excessive production of durables leads to inflation, which prompts the monetary policymaker to raise interest rates. Thus we see that both demand and supply shocks lead to changes in relative prices, which (with sticky prices) generate inefficient sectoral fluctuations.

Figures 2 and 3 show the same impulse responses with $\varphi = 0.3$ and $\varphi = 0.6$ respectively. The addition of durable adjustment costs makes a quantitative difference in that it reduces the response of sectoral dislocations by reducing the responsiveness of durable good production to shocks. However, the general qualitative character of the responses is unchanged.

Next we examine the consequences of monetary shocks of various kinds. These are shown in figure 4. Since none of these shocks affect the flexible price model, I show only the sticky price model, and thus can depict all three levels of adjustment costs in the same figure. The first column shows the consequences of a contemporaneous monetary policy shock, i.e. a

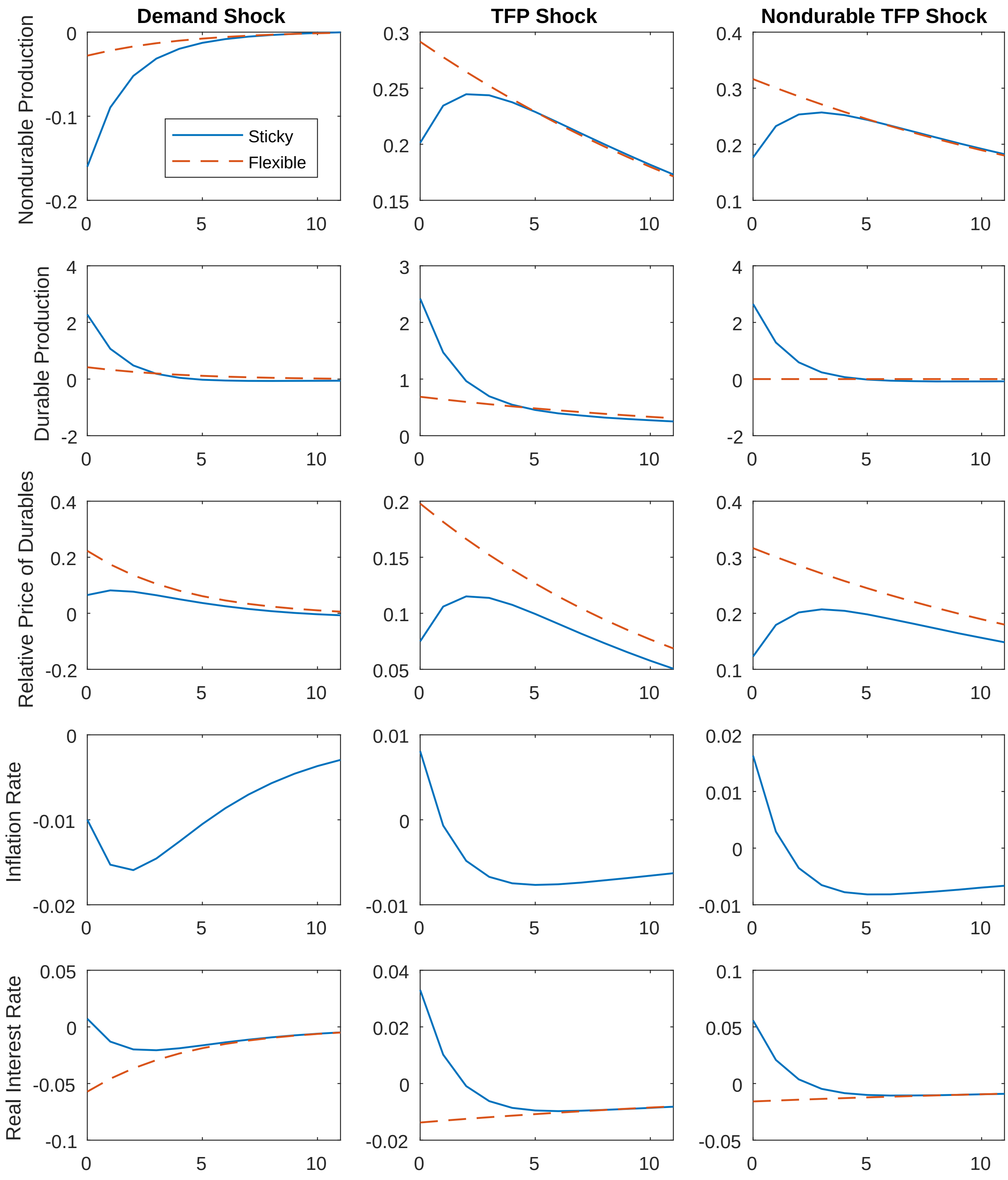

Figure 1: Shocks to Select Real Variables: No Adjustment Costs

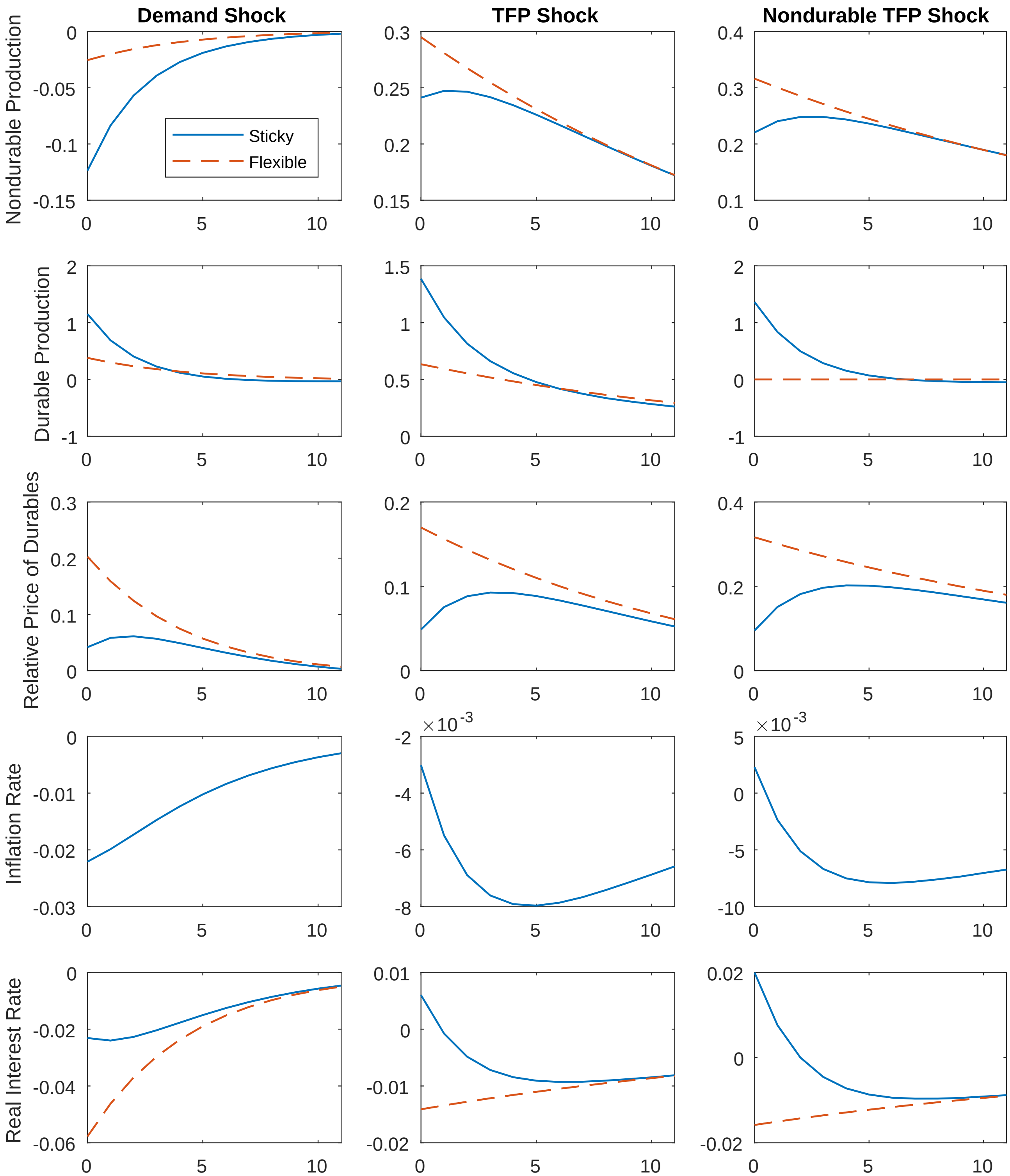

Figure 2: Shocks to Select Real Variables: Moderate Adjustment Costs ($\varphi = 0.3$)

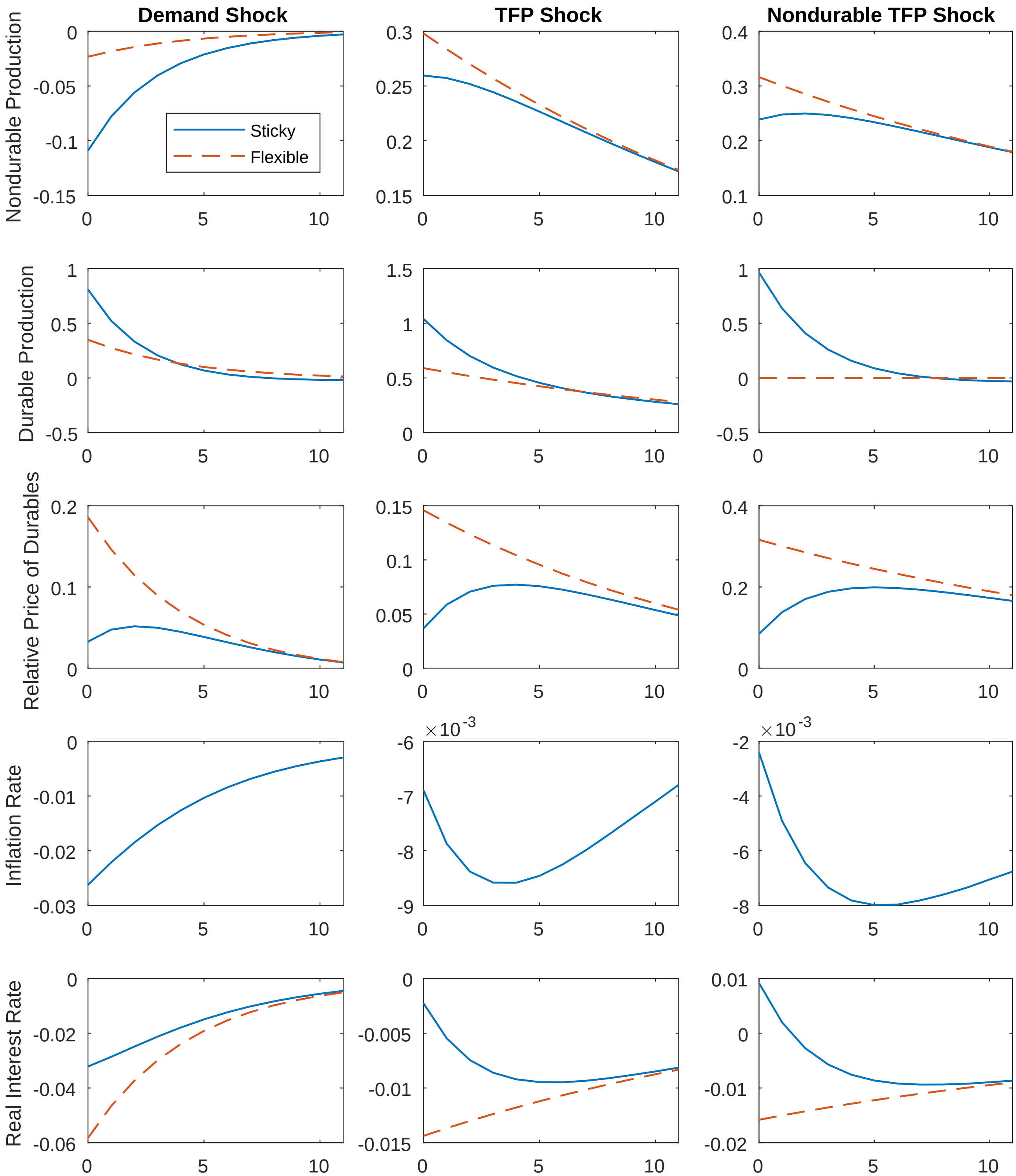

Figure 3: Shocks to Select Real Variables: High Adjustment Costs ($\varphi = 0.6$)

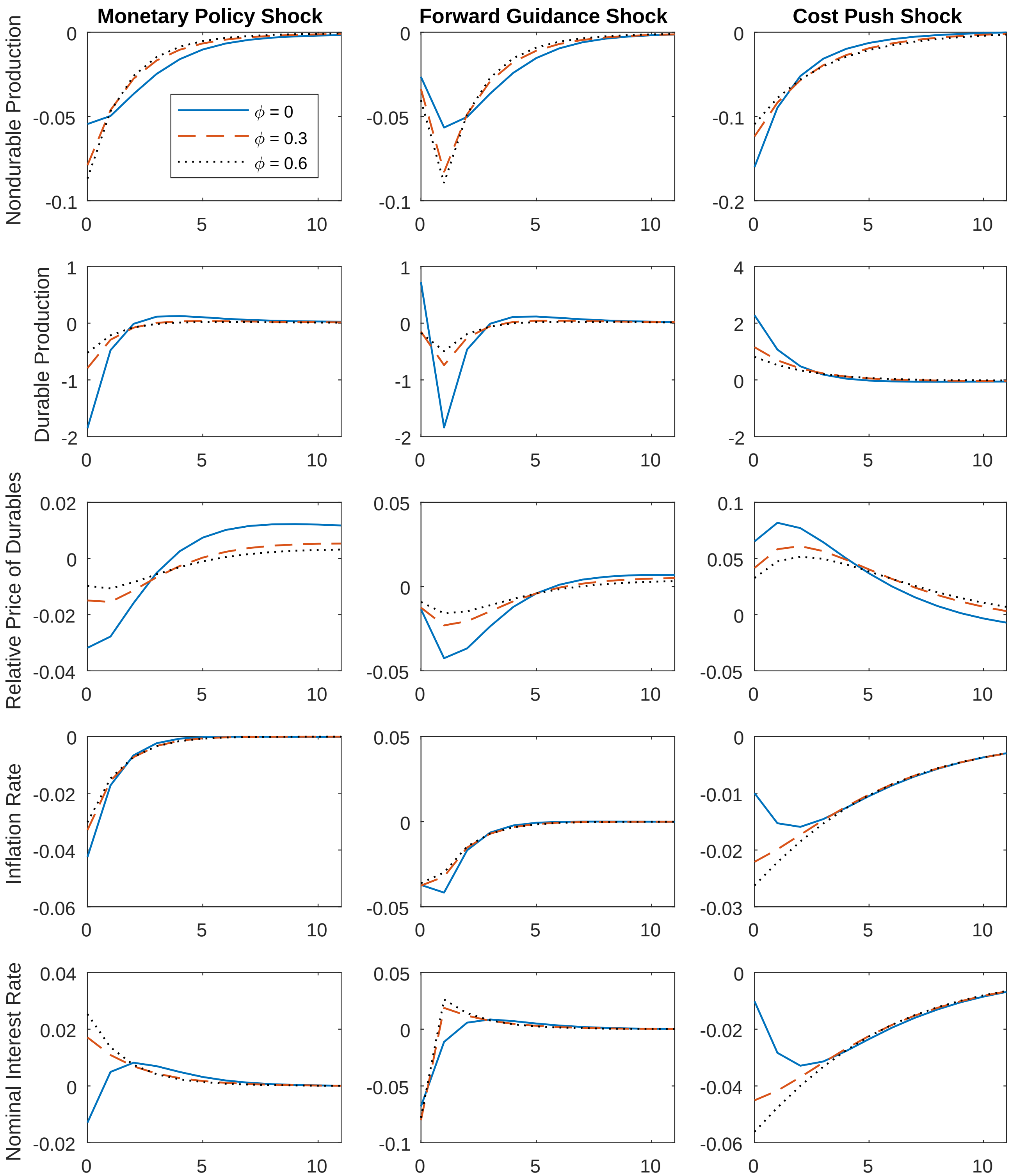

Figure 4: Shocks to Select Monetary Variables

temporary innovation to ε in equation (18).[14] The monetary policy shock causes an increase in the interest rate, which depresses demand in both sectors; however, it has a much greater effect on the durable good sector: about 33$\times$ greater in the model without adjustment costs. This excessive fall in durable production causes deflation, which prompts the monetary authority to lower nominal interest rates – indeed, nominal interest rates fall on impact in the model without adjustment costs, although the real interest rate rises. Adjustment costs mitigate the quantitative magnitude of these results, but do not change their qualitative character.

The central column of figure 4 shows a forward guidance shock, that is an innovation to ν in equation (18). Since a shock to ν_t affects the time $t+1$ interest rate, this means that time t households learn about higher interest rates in the following period. The response is actually a slight initial *increase* in durable good production. This is due to the asymmetric sensitivity of durable good demand to *current* as opposed to *future* interest rates. The expected future increase in interest rates reduces demand for durable goods substantially in the next period, and leads current demand to decline as well (through the usual forward guidance / consumption-smoothing mechanism). This reduction in current demand lowers inflation, which causes the monetary authority to reduce *current* interest rates (following the Taylor rule). Given the high sensitivity of durable demand to current interest rates, this is sufficient to raise current demand for durables. This reversal doesn't occur in the models with adjustment costs, although the basic pattern of reduced immediate response remains.

Finally, the third column of figure 4 shows the result of a cost-push shock. This shock leads to a decline in inflation, which causes the monetary authority to cut interest rates in response. This stimulates production in both sectors, but especially in the durable sector. The general result is once again excess sectoral volatility.

Interest Elasticities. Given the prominent role played by sectoral interest elasticities in the optimal policy analysis in section 2, a natural question is what these elasticities are in the quantitative model. While it's difficult to infer a single interest elasticity from the impulse responses shown in figure 4, we can measure the relative sensitivity of durable goods to monetary policy shocks by comparing the peak deviation of durable good production to the peak deviation of nondurable production. The former is a particularly useful benchmark since the baseline calibration assumes $\sigma_c = 1$, and therefore the interest elasticity of nondurable demand should be 1.

The relative magnitudes are reported in table 3, for both sectoral production and sectoral inflation. Overall, durable good production is much more sensitive to interest rates, ranging

[14] I assume the shock is AR(1) with persistence parameter 0.5.

φ	Output ε	Output ν	Inflation ε	Inflation ν
0.00	33.98	32.50	1.83	1.76
0.30	10.04	8.92	1.48	1.35
0.60	6.01	5.51	1.34	1.26

Table 3: Relative Magnitudes of Peak Deviation Following Monetary Shock.

from 30 times for no adjustment costs, to 6 times for $\varphi = 0.6$. The first is well outside the range of empirical estimates; however, the cases with adjustment costs are much more reasonable, particularly for housing. For example, while Mankiw (1985) estimates a nondurable interest elasticity of 0.5, he estimates an interest elasticity of 3.4 for the stock of durables, and 13.6 for the flow. Barsky et al. (2003) find that, following a Romer date, housing starts fall by approximately 33%, residential investment by 22%, automobile sales by 25%, and durables purchases by 12.5%; real GDP does not fall at all, though it declines relative to trend by 6%.[15] Similarly, Boivin et al. (2010) find a limited response of consumption to a monetary policy shock, while there is a large effect on industrial production and housing starts. Erceg and Levin (2006) estimate that "[A] monetary policy shock causes a decline in our broad measure of consumer durables spending that is over three times as large as for the other GDP components."

Interestingly, despite the significant difference in the timing of the sectoral response to current vs. future interest rates, the bottom line result is little different: the relative peak response of durable output to a forward guidance shock is about as large as the response to current interest rates. What changes is the timing of this reaction: it occurs in the period in which the interest rate is adjusted, rather than in the period in which households learn about the shock. Sectoral inflation is also differentially sensitive to interest rates, with durable inflation responding 83% more to a current monetary shock shock than nondurable inflation in the model without adjustment costs, and 48% more in the model with moderate adjustment costs. Note that differential inflation is not necessarily bad, since is the mechanism by which the economy adjusts relative prices, and thus reduces the differential response of output.

3.4 Comparison of Policy Rules without Adjustment Costs

I now turn to a comparison of the performance of various policy rules. I first focus on the case without adjustment costs, since this is most similar to the theoretical model developed

[15]This analysis was presented in the (2003) NBER working paper by this name. A revised version was published as Barsky et al. (2007), but the empirical analysis cited here was not present in the published version.

Policy Rule	Parameters	Welfare
Taylor Rule #1	Π: 1.5, $\tilde{y}$: 0.1	-5.76
Taylor Rule #2	Π: 2, $\tilde{y}$: 0.1	-3.66
Taylor Rule #3	Π: 2.5, $\tilde{y}$: 0.1	-2.87
Nominal Wage Target	–	-32.2
NGDP Growth Target	–	-3.05
Inflation Target	–	-1.11
Optimal Aggregate Target	Π: 0.961, $\tilde{y}$: 0.046, w: -0.007	-1.07
Sectoral Inflation Target	Π^d: 0.372, Π^c: 0.628	-0.799
Optimal Sectoral Target	Π^c: 0.627, Π^d: 0.373, $\tilde{y}$: 0.002, w: -0.003	-0.797
Lag Sectoral Inflation Target	Π^c: 0.599, Π^d: 0.401, Π^c_{t-1}: 0.215, Π^c_{t-2}: 0.154, Π^c_{t-3}: 0.063, Π^d_{t-1}: 0.0002, Π^d_{t-2}: -0.0007, Π^d_{t-3}: -0.0003	-0.712

Table 4: Comparison of Policy Rules: no adjustment costs

in section 2. Section 3.5 analyzes how the basic results of this section are changed by the inclusion of adjustment costs.

In order to compare policy rules, we must select a measure of performance. I take performance to be the unconditional expectation of household flow utility in the stationary distribution of the model, i.e.:

$$\mathcal{W} = \mathbb{E}_0 \left[\theta_t \left\{ \frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \chi_t \frac{d_t^{1-\sigma_d}}{1-\sigma_d} - \psi_t \frac{n_t^{1+\zeta}}{1+\zeta} \right\} \right]$$

This approach, in this context most closely associated with Rotemberg and Woodford (1997), is commonly used in comparing the performance of monetary policy rules in quantitative models, including the closely related papers of Mankiw and Reis (2003), Erceg and Levin (2006), Barsky et al. (2016), and Basu et al. (2016). However, this approach presents some conceptual difficulties, since it does not correspond to optimal policy in the sense used in sections 2.3 and 2.4. This should be borne in mind in the following discussion.

Aggregate Policy Rules. Table 4 compares the performance of a number of policy rules. Welfare is reported in the third column. It is calculated relative to the flexible price model, and the number reported is basis points of steady state non-durable consumption c^*. First are three Taylor rules, which all react to the output gap with a parameter of 0.1, and to inflation with parameters of 1.5, 2, and 2.5 respectively. The first is a common specification, and the middle corresponds to the baseline Taylor rule used to generate the moments and impulse responses reported in section 3.3. The welfare losses under the baseline Taylor rule are 3.66 basis points of consumption on average.[16] Increasing the weight on inflation raises welfare substantially: welfare losses are 57% greater under the lower reaction rule, and 22% less under the high reaction rule.

Aggregate Policy Rules. I next report the performance of three simple aggregate targeting rules. Targeting nominal wages performs quite poorly, producing welfare losses nearly 9x greater than the baseline Taylor rule. Targeting nominal GDP growth, which has the advantage over the Taylor rules that one need not estimate the output gap, performs 17% better than the baseline Taylor rule, and only a little worse than the strong reaction Taylor rule. However, inflation targeting is the clear winner among simple targeting rules, with welfare losses of only 1.11 basis points, 30% of the baseline Taylor rule.

I next ask whether it is possible to improve on inflation targeting. I search for the optimal target that is a weighted average of inflation, the output gap, and real wages. I find that the optimal policy places the majority of weight (96%) on inflation, with a small weight (5%) on the output gap, and a slight *negative* weight (less than 1%) on wages. The welfare gains over pure inflation targeting are marginal, reducing the welfare loss by less than 4%. Thus I conclude that if one sets policy based on aggregate variables alone, inflation targeting is a very good policy, and only marginal gains can be obtained by considering other variables.

Sectoral Policy Rules. Motivated by the optimal policy analysis undertaken in section 2, I next consider policy rules include sectoral inflation. I choose to focus on sectoral inflation both because inflation is a natural and standard choice of policy target, and because in a standard Calvo model inflation generates welfare losses directly, and corresponds to labor wedges.

I first explore a policy of sectorally weighted inflation targeting – i.e. targeting inflation in a price index that weights goods by something other than their GDP share. I search for the optimal relative weights on the two sectors, and find that it is optimal to place about a 37% weight on durable good inflation, which is very close to 3x as large as its 12.5% GDP share. This implies a ratio of about 1.69 between nondurable and durable goods, compared with a ratio of 7 going by GDP shares alone; in the language of the static optimal policy condition given in Proposition 1, replacing sectoral labor wedges with sectoral inflation rates, this corresponds to a ratio of $\varepsilon^d/\varepsilon^c$ of about 4.1. Admittedly, this is about 8x less than the ratio implied by the relative interest sensitivities of the sectors given in table 3. The welfare gain from utilizing sectoral inflation targeting is significant in relative terms, as it reduces

[16] To put this number into context, total personal consumption expenditures less durable goods in the US were about $12.5 trillion dollars in 2018, so 3.7 basis points of consumption would be about $ 4.6 billion.

the welfare loss from price stickiness by 28% relative to inflation targeting.

There is little gain from considering other aggregate variables when setting a sectoral inflation target: allowing the monetary authority to target a weighted average of sectoral inflation, plus output gap and real wage, results in a minimal gain in welfare and negligible weights on other aggregate variables.

A Lagged Sectoral Inflation Target. Finally I investigate whether there is any rule for the routine use of forward guidance, given its potential to reduce sectoral volatility as discussed in section 2.4. I implement such a policy by allowing the monetary authority to construct an inflation target that depends on both current and *lagged* values of sectoral inflation. Intuitively, forward guidance implies that at time t the monetary authority commits to some future interest rate movement. This implies that future policy depends on the current state in some way, or alternatively that *current* policy depends on the *past* state.

The results are shown in the last row of table 4. The weights are reported such that the weights on *current* inflation sum to 1, whereas the weights on lagged inflation are reported relative to these weights. The optimal rules weights durable vs. nondurable inflation at about 60/40, implying a somewhat higher weight on current durable inflation than the pure sectoral inflation target. Optimal policy implies a negligible weight on lagged durable inflation, but a substantial weight on lagged nondurable inflation: the cumulative weight on the last three quarters of lagged inflation is about 0.44, or somewhat higher than the weight on current durable inflation.[17] This policy results in a small but significant welfare gain relative to the sectoral inflation target — the welfare loss from price stickiness is about 11% less.

One way to think about this policy rule is that past nondurable inflation leads the monetary authority to raise or lower its current inflation target, where the current target is itself a weighted average of nondurable and durable inflation. For example, if the last three quarters had each seen 1% nondurable inflation, the rule above would imply that the current inflation target should be about −0.43% instead of 0%. Intuitively, past inflation indicates excessive stimulation of the nondurable sector. This policy rule implies that in those earlier periods the Fed responded by promising to target a lower rate of inflation in the future, thereby promising higher future interest rates. This would have reduced past inflation rates through forward guidance, without excessively stimulating the durable sector, as would be the case if the interest rates had been immediately adjusted. This also explains why it is past nondurable inflation in particular that the rule responds to.

Of course, this reduction in the inflation target is unlikely to be optimal ex post. This policy requires commitment. The benefit derives not from the current change in the inflation

[17] An earlier version of this paper mistakenly reported that these weights were on lagged durable inflation.

target, but in the expectation of such future changes. This allows the central bank to make greater use of forward guidance in setting policy, and thus utilize current interest rates less. Whether committing to such a policy is feasible is beyond the scope of this paper.

3.5 Consequences of Adjustment Costs

I now turn to comparing policy rules in the presence of adjustment costs. Motivated by the results above, I focus on just three policy rules: inflation targeting, weighted sectoral inflation targeting, and lag weighted sectoral inflation targeting. In the last case I only allow for lags in nondurable inflation.

The results are shown in figure 5. Panel (a) shows the absolute welfare loss under inflation targeting (IT), weighted inflation targeting (WIT), and Lag weighted inflation targeting (LWIT) for varying levels of the adjustment costs parameter φ. Panel (b) shows the optimal weight on the durable sector under WIT and under LWIT, along with the cumulative optimal weight on lagged nondurable inflation under LWIT. Unsurprisingly, higher adjustment costs imply lower overall welfare losses (since adjustment costs reduce aggregate volatility). Moreover, since higher adjustment costs reduce the relative sensitivity of durable goods to interest rates, they imply a lower optimal weight on both sectoral and lagged inflation, as well as lower relative gains in welfare from sectoral and lag policy rules. However, the same qualitative ordering across the three cases is maintained. Notably, the optimal weight on durable sector inflation remains well above the GDP share of 0.125 even for quite large levels of adjustment costs. Moreover, the weight on cumulative past nondurable inflation remains fairly large (above the weight on durable goods) even for high levels of adjustment costs.

Panels (c) and (d) explore how the relative interest-sensitivity of durable goods varies with φ, and how the optimal weight on durable inflation under WIT varies with relative interest sensitivity. As before, interest sensitivity of the durable sector is calculated as the peak response of durable production to a monetary policy shock divided by the peak response of nondurable production, calculated from the monetary policy IRF under the baseline Taylor rule. This is now related to the relative optimal weight on durable goods, i.e. the optimal weight on durables divided by the optimal weight on non-durables, multiplied by 7. Thus a value of 1 corresponds to weighting sectoral inflation by their GDP shares, i.e. inflation targeting.

3.6 Alternate Specifications and Robustness Checks

I now present a number of robustness checks and alternate specifications. These are shown in table 5.

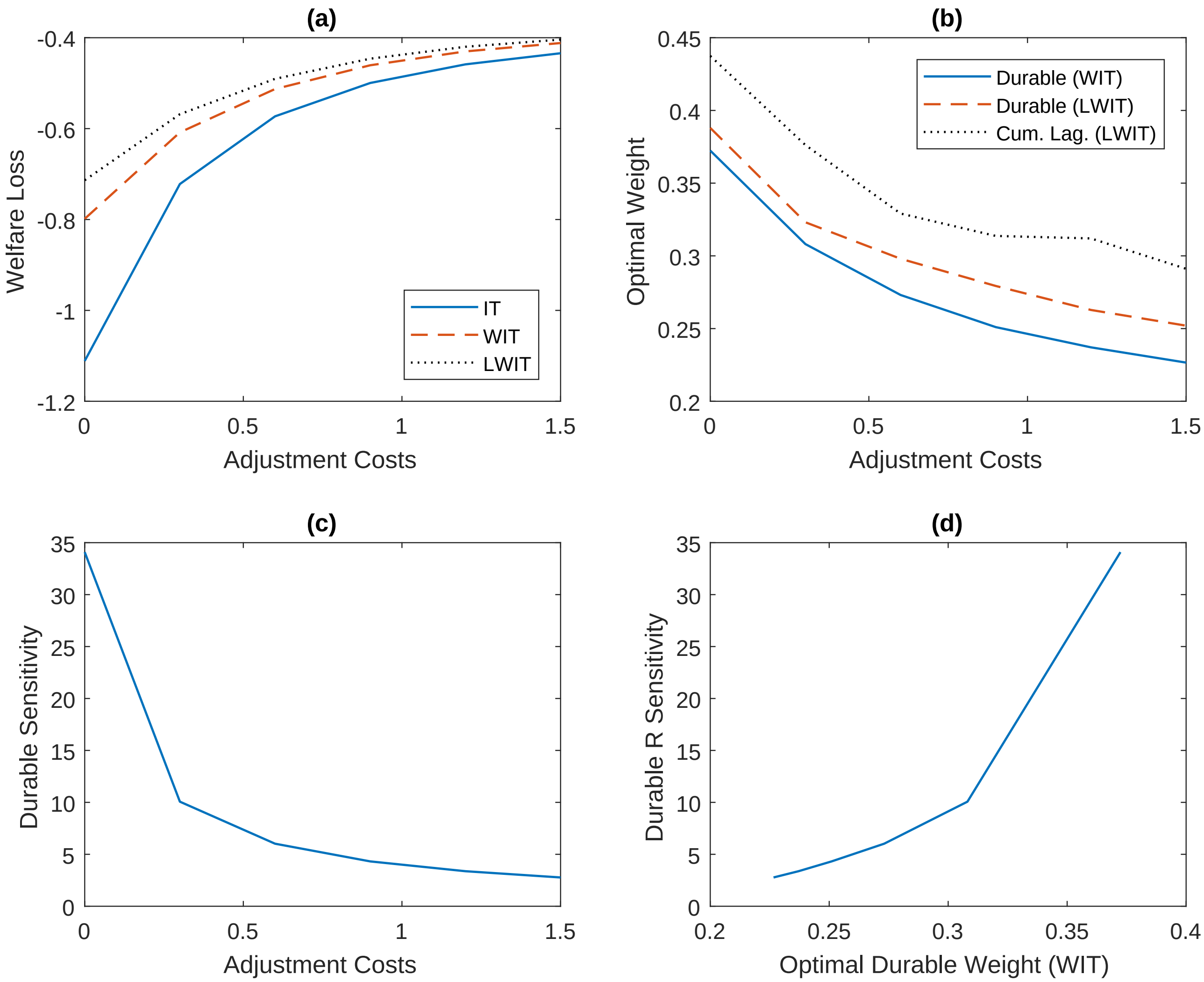

Figure 5: Optimal Rules under Adjustment Costs

Case	d-Sensitivity	Welfare IT	Welfare WIT	Weight Π^d	d GDP share
Seed = 0 (Bench)	34.0	-1.11	-0.80	0.372	0.125
Seed = 10	33.6	-1.31	-0.97	0.388	0.125
Seed = 20	34.1	-1.96	-1.68	0.354	0.125
Seed = 30	32.7	-2.06	-1.77	0.357	0.125
θ only	34.9	-2.01	-1.20	0.447	0.125
μ only	33.9	-2.19	-1.89	0.653	0.125
z only	33.8	-1.62	-1.18	0.347	0.125
z_c only	33.8	-4.92	-3.82	0.345	0.125
δ=1	1.00	-1.57	-1.57	0.504	0.500
δ=1, $\alpha_d = 2/3$	1.12	-1.95	-1.69	0.722	0.500
δ=1, $\phi_d = 0.9$	1.22	-2.11	-1.54	0.847	0.500
δ=1, $\sigma_d = 1/3$	2.86	-1.47	-1.40	0.448	0.394

Table 5: Alternate Specifications and Robustness Checks

Random Seed. Since expected welfare is calculated along a particular simulated path, it is necessary to fix the random seed when comparing policy rules, so that the comparison is made with respect to the same set of shocks. However, since the shocks experienced along any particular simulated path will differ, the relative welfare and optimal policy weights may differ depending on the random seed chosen. To test whether the results are robust to the choice of random seed, I repeat the main analysis using several random seeds. The key results are shown in the first four rows of table 5. While the precise numbers calculated change somewhat, the main qualitative conclusions remain: it is optimal to give durable inflation greater weight in the inflation target, and this represents a sizeable increase in welfare.

Variance of Shocks. I next explore which shocks are driving the policy results. To do this I alter the variance of the shocks by reducing the variance of all shocks but one to 10^{-6}, and increase the variance of one shock to 10^{-4}. This makes a single shock about 100 times more variable than the others, and thus responsible for the great majority of the variation.

The results are shown in the middle rows of table 5 for four shocks: the demand shock θ, the cost-push shock μ, the aggregate TFP shock z, and the nondurable sector TFP shock z_c. In all four cases it is optimal to weight durable inflation at a higher rate than its GDP share. The optimal weight is highest for the cost-push shock case, and next highest for the demand shock, with the aggregate and sectoral TFP shocks being overall less responsible for a greater weight on the durable sector.

Alternate Sources of Differential Interest Sensitivity. Thus far we have only looked at the consequences of differential sectoral interest sensitivity arising from differential durability of goods. This is doubtless a significant source of differential sensitivity in practice; however, other sources matter as well. The optimal policy results derived in section 2 did not rely exclusively on durability, but allowed for other sources of differential sectoral sensitivity. I therefore examine whether alternate sources of differential sensitivity produce similar results in the quantitative model.

I make two changes to the model: first I set $\delta = 1$, so that the d-good now experiences full depreciation and is therefore a non-durable good. Next I set $\chi = 1$, so that the steady state GDP share of each good is 1/2. Thus we are considering an economy with two equally-sized sectors, both of which are nondurable. I then allow these d-sector to differ with respect to capital share α_d, price-stickiness ϕ_d, and risk aversion σ_d, varying only one parameter at a time. The results are presented in the final rows of table 5. Since varying parameters also varies the sectoral GDP shares, I also report the steady state GDP share of the d-sector under this specification.

I next inquire whether the results are similar with alternative sources of differential interest sensitivity. For this analysis, I assume full depreciation of the good d_t, i.e. $\delta = 1$, so that both goods are nondurable. I adjust the parameters so that in the symmetric case the GDP share of the d sector is 1/2. I then let other parameters differ between the two sectors. I report the steady state GDP share of the d-sector in each case, since the steady state changes when other parameters are changed. To verify that everything is working properly, I first report the result of the baseline model without depreciation and equally-sized sectors. As expected, the optimal weight on the d-sector is very close to the GDP share of 1/2, and weighted inflation targeting produces the same welfare as baseline inflation targeting.[18] I next alter other sectoral parameters one at a time.

I first consider the effect of a higher capital share in one sector, in this case the d-sector. A higher capital share does not directly affect interest sensitivity; however, it does affect the connection between output and labor costs (and therefore inflation). A higher capital share implies stronger concavity of the production function with respect to labor, and therefore a given swing in demand prompts a larger swing in labor costs. This increases the volatility of the given sector. In this case, this should make it optimal to respond more to fluctuations in inflation in the d-sector, since these correspond to larger inefficiencies in this sector. However, the d-sector is not much more sensitive to interest rates than the c-sector.

[18] The optimal weight on the durable sector differs slightly from the GDP share simply because the comparison is made along a particular simulated path, which may be subject to certain shocks that favor weighting one sector over the other.

I next consider the result of differential price stickiness between the two sectors. This has been a commonly-considered reason for differential interest-sensitivity, for example by Mankiw and Reis (2003). Generally speaking, the sector with stickier prices should be weighted more heavily, since output in that sector is more sensitive to interest rates, and fluctuations in output (and therefore labor wedges) are greater for a given perceived sectoral inflation rate. The results are consistent with these predictions. Perhaps surprisingly, the d-sector is not much more sensitive to a monetary policy shock. However, it is optimal to respond much more to inflation in this sector than implied by its GDP share.

Finally I consider the effect of a higher intertemporal elasticity of substitution in the d-sector. In particular, I set $\sigma_d = 1/3$, implying that the d-sector is about three times more sensitive to interest rates than the c-sector. This is as close as one can get to simply changing interest elasticity directly. The differential sensitivity matches closely: it is a bit less than 3, since some relative price adjustment takes place, and so not all of the effect of the monetary policy shock is felt on quantities. Nevertheless, it is optimal to place a greater weight on inflation in the d-sector.

4 Conclusion

The results of this paper suggest that monetary policymakers should give more consideration to differential sectoral interest sensitivity when setting policy. In particular, policymakers should weight sectors by their interest elasticity, should take dynamic demand effects from durable goods into account when setting policy, and should systematically utilize forward guidance in the conduct of monetary policy.

Several areas of future work remain. The analysis in this paper highlights the importance of sectoral interest elasticities for monetary policy, yet relatively little empirical work focuses on estimating these elasticities in a consistent way. Further, the model implies that durable goods are relatively more sensitive to current interest rates than to future rates, implying that forward guidance is a useful tool to reduce sectoral volatility; yet I am aware of no work that estimates sectoral elasticities with respect to current vs. future interest rates. Finally, the results imply limitations on the ability of monetary policy alone to stabilize the economy. This suggests that other policies might be useful in supplementing monetary policy, particularly if these policies have sectoral effects that differ from monetary policy.

References

Luis J Álvarez, Emmanuel Dhyne, Marco Hoeberichts, Claudia Kwapil, Hervé Le Bihan, Patrick Lünnemann, Fernando Martins, Roberto Sabbatini, Harald Stahl, Philip Vermeulen, et al. Sticky prices in the euro area: A summary of new micro-evidence. *Journal of the European Economic Association*, 4(2-3):575–584, 2006.

Kosuke Aoki. Optimal monetary policy responses to relative-price changes. *Journal of monetary economics*, 48(1):55–80, 2001.

Robert Barsky, Christopher L House, and Miles Kimball. Do flexible durable goods prices undermine sticky price models? Technical report, National Bureau of Economic Research, 2003.

Robert Barsky, Christopher L House, Christoph Boehm, and Miles S Kimball. Monetary policy and durable goods. 2016.

Robert B Barsky, Christopher L House, and Miles S Kimball. Sticky-price models and durable goods. *The American economic review*, 97(3):984–998, 2007.

Susanto Basu, Pierre De Leo, et al. *Should central banks target investment prices?* Boston College, Department of Economics, 2016.

Paul Beaudry, Dana Galizia, and Franck Portier. Reconciling hayek's and keynes' views of recessions. *The Review of Economic Studies*, 85(1):119–156, 2017.

Pierpaolo Benigno. Optimal monetary policy in a currency area. *Journal of international economics*, 63(2):293–320, 2004.

Mark Bils, Peter J Klenow, and Benjamin A Malin. Testing for keynesian labor demand. *NBER Macroeconomics Annual*, 27(1):311–349, 2013.

Jean Boivin, Michael T Kiley, Frederic S Mishkin, et al. How has the monetary transmission mechanism evolved over time? *Handbook of Monetary Economics*, 3:369–422, 2010.

Hafedh Bouakez, Emanuela Cardia, and Francisco J Ruge-Murcia. The transmission of monetary policy in a multisector economy. *International Economic Review*, 50(4):1243–1266, 2009.

Hafedh Bouakez, Emanuela Cardia, and Francisco J Ruge-Murcia. Durable goods, intersectoral linkages and monetary policy. *Journal of Economic Dynamics and Control*, 35 (5):730–745, 2011.

Alessandro Cantelmo and Giovanni Melina. Monetary policy and the relative price of durable goods. *Journal of Economic Dynamics and Control*, 86:1–48, 2018.

Carlos Carvalho et al. Heterogeneity in price stickiness and the real effects of monetary shocks. *The BE Journal of Macroeconomics*, 6(3):1–58, 2006.

Lawrence J Christiano, Martin Eichenbaum, and Charles L Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1): 1–45, 2005.

Christopher Erceg and Andrew Levin. Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53(7):1341–1359, 2006.

Robert G King and Sergio T Rebelo. Resuscitating real business cycles. *Handbook of macroeconomics*, 1:927–1007, 1999.

N Gregory Mankiw. Consumer durables and the real interest rate. *The Review of Economics and Statistics*, pages 353–362, 1985.

N Gregory Mankiw and Ricardo Reis. What measure of inflation should a central bank target? *Journal of the European Economic Association*, 1(5):1058–1086, 2003.

Alisdair McKay and Johannes F Wieland. Lumpy durable consumption demand and the limited ammunition of monetary policy. 2019.

Matthew Rognlie, Andrei Shleifer, and Alp Simsek. Investment hangover and the great recession. *American Economic Journal: Macroeconomics*, 10(2):113–53, 2018.

Julio J Rotemberg and Michael Woodford. An optimization-based econometric framework for the evaluation of monetary policy. *NBER macroeconomics annual*, 12:297–346, 1997.

Frank Smets and Rafael Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606, 2007.

John B Taylor. Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier, 1993.

A Adjustment Costs and Habit Formation in Model without Inflation

The quantitative model presented in section 3 included adjustment costs. However, the analytical results in section 2 did not include adjustment costs. This section extends the analysis in section 2 to include adjustment costs and habit formation of various kinds.

Consider the model analyzed in section 2, but with household utility:

$$\sum \beta^t \theta_t \left[u\left(\vec{c}_t, \vec{c}_{t-1}\right) - v\left(n_t\right) \right]$$

This specification nests various common forms of internal habit formation and consumption adjustment costs, as long as they affect the utility function rather than the budget constraint.[19] Household optimality conditions are just as in the baseline model except that the good j pricing equation is:

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \frac{\left(u_{t+1}\right)_{c_t^j}}{R_{t+1}\lambda_{t+1}} + \frac{1-\delta^j}{R_{t+1}} p_{t+1}^j$$

The only difference relative to the baseline case is the inclusion of the term $\left(u_{t+1}\right)_{c_t^j} / \left(R_{t+1}\lambda_{t+1}\right)$ in the good j pricing equation. This term, which will generally be negative, reflects the cost of higher past consumption in the following period. Note that the inclusion of adjustment costs might also affect the value of marginal utility of consumption. Firm optimality conditions and market clearing expressions are just as in the baseline model.

My first result is that the expressions that characterize optimal policy in the one-period fixed price case and the N-period fixed price case without commitment do not change.

Proposition 8 (Optimal Policy without Commitment and Adjustment Costs)**.** *With adjustment costs and N-period fixed prices, the optimal policy without commitment satisfies:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}}\right) \tau_{t+1}^j$$

With $N = 1$, this becomes:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

Proof. See appendix D. □

Intuitively, the only effect of adjustment costs is to change the magnitudes of the interest

[19] Thus it differs from the form of adjustment cost used in the quantitative model in section 3.

elasticities and the precise values of the labor wedges. The static sectoral tradeoff and the effect of durable overhang are fully summarized by the same expressions.

Matters change a bit with commitment. The best we can manage is the following:

Proposition 9 (Optimal Policy with Commitment and Adjustment Costs)**.** *With adjustment costs and N-period fixed prices, the optimal policy with commitment satisfies:*

$$\sum_{t=0}^{N-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_k}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

is the overhang-augmented labor wedge.

Proof. See appendix D. □

This expression is as in Proposition 5 except the sum is up to $t = N - 1$, rather than $t = k$. This is because, with the inclusion of lagged consumption in utility, it is no longer the case that past interest rates have no effect on future consumption. Moreover, we can no longer simplify the expression to apply to two periods only, since the effect of forward guidance is no longer the same for all interest rates after the current period. This is simply because the equations for period t demand depend on $\vec{c}_{t-1}$ and $\vec{c}_{t+1}$, and thus may depend in an arbitrary fashion on all past and future interest rates.

B Uncertainty

The baseline model in section 2 assumed certainty. This was a convenient assumption to obtain simple optimal policy expressions, but raises the question whether the general results are sensitive to this assumption. This section extends the results to the stochastic case, and shows that the general character of optimal policy does not change.

Flexible Prices. Consider the model in section (2.1) with the inclusion of uncertainty. In particular, I suppose that the demand shock θ is stochastic, and also allow parameters of the production function to be stochastic.[20] I continue to assume that the real bond is safe,

[20]Since the firm production functions given in (6) were time dependent, they already allowed for predictable change in production parameters such as productivity or the shape of the production function. We are now allowing these changeds to be stochastic.

so that R_{t+1} is known at time t. Under these assumptions, the household budget constraint is unchanged, and the household objective function becomes:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \theta_t [u(\vec{c}_t) - v(n_t)]$$

The household optimality expressions are:

$$\frac{v_{n_t}}{\lambda_t} = w_t \tag{19}$$

$$1 = \mathbb{E}_t \left[\frac{\beta \theta_{t+1}}{\theta_t} \frac{\lambda_{t+1}}{\lambda_t} R_{t+1} \right] \tag{20}$$

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + (1 - \delta^j) \mathbb{E}_t \left[\frac{\beta \theta_{t+1}}{\theta_t} \frac{\lambda_{t+1}}{\lambda_t} p_{t+1}^j \right] \tag{21}$$

To see what difference uncertainty makes in the demand equations, we combine the good j demand equation with the Euler equation to obtain:

$$p_t^j = \frac{u_{c_t^j}}{\lambda_t} + \left(\frac{1 - \delta^j}{R_{t+1}} \right) \frac{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} p_{t+1}^j \right]}{\mathbb{E}_t \left[\theta_{t+1} \lambda_{t+1} \right]}$$

Under certainty, the term $\theta_{t+1}\lambda_{t+1}$ on the right-hand side cancel out, whereas with uncertainty they do not. Thus there is an additional effect due to covariance between future prices and the future marginal utility of consumption.

One-period fixed prices. We now turn to optimal policy in the case of one-period fixed prices. The optimal policy expression turns out to be just as in the case with certainty:

Proposition 10 (One-period fixed prices with uncertainty)**.** *With one-period fixed prices and uncertainty, the optimal policy is to set the interest rate so that:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

Proof. See appendix D. □

Intuitively, the inclusion of uncertainty makes no difference to optimal policy under one-period fixed prices because optimal tradeoff between sectoral production in the current period is unaffected by uncertainty. Effects of future uncertainty are captured in the equilibrium prices, which are flexible and therefore respond optimally to changes in the covariance arising from changes in current production.

N-period fixed prices without commitment. We next suppose that prices are fixed for multiple periods, and the monetary authority lacks commitment. In this case the policy rule is quite similar to the case with certainty, with the only difference deriving from covariance between future wedges and future marginal utility. The following proposition gives the optimal policy expressions:

Proposition 11 (N-period fixed prices with uncertainty, no commitment)**.** *With N-period fixed prices, no commitment, and uncertainty, the optimal policy is to set the interest rate so that:*

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}}\right) \frac{\mathbb{E}_t\left[\tau_{t+1}^j \theta_{t+1}\lambda_{t+1}\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}$$

Proof. See appendix D. □

Recall that when $\delta^1 = 1$, so that good 1 is a nondurable good, $\lambda_t = u_{c_t^1}$. This expression differs from the case with certainty by the inclusion of the covariance between the future labor wedge τ_{t+1}^j and the future marginal utility of consumption $\theta_{t+1}\lambda_{t+1}$. A positive covariance in sector j implies that production in this sector is relatively low when aggregate consumption is low, in other words the output gap in this sector is procyclical. This makes the effects of durable overhang greater.

Uncontingent commitment. We next consider cases where the central bank is able to commit to a future interest rates. With commitment, we must make a further distinction: that the central bank can commit to a future path of interest rates, or that it may commit to a future path of *state contingent* interest rates. I start with the former.

Suppose that the monetary authority can commit to a particular path of future interest rates, but cannot commit to a state-contingent rate. This may be because the state is partially unobservable to market participants, and thus only an announced future rate allows the monetary authority to maintain its reputation. Then it is unclear whether the monetary authority will prefer to commit or to retain the flexibility to respond to future shocks. Suppose that the monetary authority commits to a particular path of interest rates over the next N periods. Thus at time 0 the central bank chooses $\{R_{t+1}\}_{t=0}^{N-1}$. The following proposition gives the expression for the optimal choice of R_k:

Proposition 12 (Optimal Policy under Uncontingent Commitment)**.** *In the problem with uncertainty and N-period fixed prices, when the central bank must choose a path of interest rates at time* 0*, the optimal choice of R_k satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{k} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_{k+1}}^{y_t^j} \chi_t^j = 0$$

where
$$\chi_t^j = \tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}$$
is the durable overhang-augmented labor wedge.

Proof. See appendix D. □

As in the certainty case, sectors are equally sensitive to all interest rates more than one period ahead.

Lemma 3 (Symmetric effects of forward guidance under uncertainty)**.** *In the model with uncertainty,* $\epsilon_{R_k}^{y_t^j} = \epsilon_{R_\ell}^{y_t^j}$ *for* $k, \ell \in [t+2, N]$*.*

Proof. See appendix D. □

Lemma 3 immediate allows us to obtain an analogous result to Proposition 6.

Corollary 1. *The optimal policy expression in Proposition 12 for* $k > 0$ *may be written:*

$$\mathbb{E}_0 \frac{y_k}{R_k} \sum_j \gamma_k^j \epsilon_{R_{k+1}}^{y_k^j} \chi_k^j = \mathbb{E}_0 y_{k-1} \sum_j \gamma_{k-1}^j \left(\epsilon_{R_k}^{y_{k-1}^j} - \epsilon_{R_{k+1}}^{y_{k-1}^j}\right) \chi_{k-1}^j$$

Proof. Take the difference between the expressions for the optimal choices of R_{k+1} and R_k in Proposition 12, and apply Lemma 3. □

Compare this expression to the optimal policy expression without commitment, which we can write as:
$$\mathbb{E}_k \sum_j \gamma_k^j \epsilon_{R_{k+1}}^{y_k^j} \chi_k^j = 0$$
The expression with commitment differs in two ways from the no commitment case in two respects. First, the right-hand side contains $\epsilon_{R_k}^{y_{k-1}^j} - \epsilon_{R_{k+1}}^{y_{k-1}^j}$. This captures the potential benefit of using forward guidance, which depends on the differential sensitivity of sectoral volatility to current and future interest rates. Second, the lefthand side contains $\mathbb{E}_0$ rather than $\mathbb{E}_k$. This captures that the monetary authority cannot make the choice of future interest rates state contingent, and thus must commit to future interest rates with the information set available at time 0, rather than at time t.

State-contingent Commitment. Now suppose the central bank can commit to a path of *state-contingent* future interest rates. Let the shock at time t be s_t, let the history of shocks from time 0 to time t by $s^t = \{s_0, \ldots, s_t\}$, and let the probability of this history be $q(s^t)$. Then at time 0, the central bank chooses:

$$\left\{R_{t+1}\left(s^t\right)\right\}_{t=0}^{N-1}$$

Let $V_N\left(\vec{c}_{N-1}, s^N\right)$ be the flexible price value function at time N. Then the objective function of the central bank is:

$$\sum_{t=0}^{N-1}\sum_{S^t} q\left(s^t\right)\beta^t\theta_t\left[u\left(\vec{c}_t\right) - v\left(n_t\left(\vec{c}_t, \vec{c}_{t-1}\right)\right)\right] + \beta^T\sum_{S^T} q\left(s^T\right)\theta_T V\left(\vec{c}_{T-1}\right)$$

where demand $c_t^j(s^t)$ may depend on the entire path of interest rates $\{R_{t+1}(s^t)\}_{t=0}^{N-1}$. Note that Lemma 1 still applies, so that demand at time t does not depend on past choices of interest rates, since current prices are fixed. This demand $c_t^j(s^t)$ depends only on $\left\{R_{t+k}\left(s^{t+k}\right)\right\}_{k=1}^{N-t}$. The following proposition gives a condition for the optimal choice of $R_{k+1}\left(s_*^k\right)$.

Proposition 13 (Optimal Policy under uncertainty with full commitment)**.** *The optimal choice of $R_{k+1}\left(s_*^k\right)$ in the N-period fixed price model with full commitment is:*

$$\sum_{t=0}^{k} q\left(s_*^t\right)\beta^t\theta_{t*}\lambda_{t*}y_{t*}\left\{\sum_j \epsilon_{R_{k+1}\left(s_*^k\right)}^{y_t^j}\gamma_{t*}^j\chi_{t*}^j\right\} = 0$$

where x_{t*} denotes $x(s_*^t)$, where s_*^t lies on the path defined by s_*^k, i.e. $s_*^k \subset s_*^t$.

Proof. See appendix D. □

To understand these expressions let's analyze the choices of particular interest rates. First the choice of R_1 yields the expression:

$$\sum_j \epsilon_{R_1}^{y_0^j}\gamma_0^j\chi_0^j = 0$$

which is just the same as in the case without commitment, since as before future demand does not depend on past interest rates.[21] Now consider the choice of $R_2(s_*^1)$. This yields:

$$\theta_0\lambda_0 y_0\left\{\sum_j \epsilon_{R_2\left(s_*^1\right)}^{y_0^j}\gamma_0^j\chi_0^j\right\} + q\left(s_*^1\right)\beta\theta_{1*}\lambda_{1*}y_{1*}\left\{\sum_j \epsilon_{R_2\left(s_*^1\right)}^{y_1^j}\gamma_{1*}^j\chi_{1*}^j\right\} = 0$$

[21]That is, Lemma 1 holds for the stochastic case.

Rearranging and taking the difference, we obtain:

$$\sum_j \epsilon^{y_1^j}_{R_2(s_*^1)} \gamma^j_{1*} \chi^j_{1*} = \frac{\theta_0 \lambda_0 y_0}{\beta \theta_{1*} \lambda_{1*} y_{1*}} \left[\sum_j \gamma^j_0 \chi^j_0 \left(\epsilon^{y_0^j}_{R_1} - \frac{\epsilon^{y_0^j}_{R_2(s_*^1)}}{q\left(s_*^1\right)} \right) \right]$$

This captures the rule of forward guidance. Note that since s_*^1 represents only a subset of states reachable from the initial period, the demand elasticity with respect to the interest rate that prevails in these states will generally be less than the demand elasticity with respect to an interest rate that holds across all states. Dividing the elasticity by $q(s_*^t)$ corrects for this factor, so that the result captures the probability-adjusted interest elasticity of demand. Thus these elasticities should be similar in magnitude, even if the state is quite unlikely to occur.

With contingent interest rates, the symmetric effect of forward guidance at various time horizons no longer holds, and thus we cannot obtain an analogous simplified expression for all future interest rates.

C Derivation of Calvo Pricing Model

Final Good Aggregator. Suppose that in sector j there is a unit interval of intermediate good firms indexed by i. The output of intermediate good firms is combined by a Dixit-Stiglitz aggregator firm to produce final goods:

$$y_t^j = \left(\int_i \left(y_{it}^j\right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

This implies demand function for intermediate goods:

$$\frac{y_{it}^j}{y_t^j} = \left(\frac{p_{it}^j}{p_t^j} \right)^{-\epsilon}$$

Combining these, we obtain an expression for the aggregate price level:

$$\left(p_t^j\right)^{1-\epsilon} = \int_i \left(p_{it}^j\right)^{1-\epsilon}$$

Intermediate Good Firms. Suppose that intermediate good firms produce using production functions:

$$y_{it}^j = z_t^j \left(n_{it}^j\right)^{1-\alpha}$$

Suppose further that firms discount *nominal* profits in time t at rate Q_t. Suppose that there is a per-unit production subsidy $1+\tau = \frac{\epsilon}{\epsilon-1}$, which is chosen to offset under production from firms' market power.

Suppose that every period a random fraction $1-\phi_j$ of firms can adjust their nominal prices, with all other firms leaving their prices unchanged. Consider the price-setting problem of a particular firm, who takes the paths of $\left\{p_t^j, y_t^j, w_t\right\}$ as given, where w_t is the *nominal* wage. Then the price-setting problem of firm i that can adjust its price in period t is to choose p_{it}^j to maximize:

$$\sum_{s=0}^{\infty} \phi_{js} Q_{t+s} \left[(1+\tau) \left(p_{it}^j\right)^{1-\epsilon} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j - w_{t+s} \left(\left(\frac{p_{it}^j}{p_{t+s}^j} \right)^{-\epsilon} \frac{y_{t+s}^j}{z_{t+s}^j} \right)^{\frac{1}{1-\alpha}} \right]$$

Again assuming $1+\tau = \frac{\epsilon}{\epsilon-1}$, this yields optimality condition:

$$\sum_{s=0}^{\infty} \phi_{js} Q_{t+s} \left(p_t^{j*}\right)^{-\epsilon} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j = \frac{1}{1-\alpha} \sum_{s=0}^{\infty} \phi_j^s Q_{t+s} w_{t+s} \left(p_t^{j*}\right)^{-\left(1+\frac{\epsilon}{1-\alpha}\right)} \left(p_{t+s}^j\right)^{\frac{\epsilon}{1-\alpha}} \left(\frac{y_{t+s}^j}{z_{t+s}^j} \right)^{\frac{1}{1-\alpha}}$$

Solving for p_t^{j*}, we obtain:

$$\left(p_t^{j*}\right)^{\left(1+\frac{\alpha\epsilon}{1-\alpha}\right)} = \frac{\sum_{s=0}^{\infty} \phi_j^s Q_{t+s} w_{t+s} \left(p_{t+s}^j\right)^{\frac{\epsilon}{1-\alpha}} \left(\frac{y_{t+s}^j}{z_{t+s}^j} \right)^{\frac{1}{1-\alpha}}}{(1-\alpha) \sum_{s=0}^{\infty} \phi_j^s Q_{t+s} \left(p_{t+s}^j\right)^{\epsilon} y_{t+s}^j}$$

We can express this as:

$$\left(p_t^{j*}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \frac{1}{1-\alpha} \frac{X_t^j}{Z_t^j}$$

where X_t^j and Z_t^j are defined recursively as

$$X_t^j = w_t \left(p_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left(y_t^j / z_t^j\right)^{\frac{1}{1-\alpha}} + \phi_j \frac{Q_{t+1}}{Q_t} X_{t+1}^j$$
$$Z_t^j = \left(p_t^j\right)^{\epsilon} y_t^j + \phi_j \frac{Q_{t+1}}{Q_t} Z_{t+1}^j$$

Aggregate price dynamics. Suppose that all firms follow the pricing rule above. Then at every point in time, we can distinguish between the aggregate price p_t^j, and the price of adjusting firms p_t^{j*}. From the expression for the price index p_t^j, the aggregate price level evolves according to:

$$\left(p_t^j\right)^{1-\epsilon} = \phi_j \left(p_{t-1}^j\right)^{1-\epsilon} + (1-\phi_j) \left(p_t^{j*}\right)^{1-\epsilon}$$

We can write this is inflation terms as:

$$\left(\Pi_t^j\right)^{1-\epsilon} = 1 + (1-\phi_j)\left[\left(\Pi_t^{j*}\right)^{1-\epsilon} - 1\right]$$

where $\Pi_t^j = p_t^j / p_{t-1}^j$ and $\Pi_t^{j*} = p_t^{j*}/p_{t-1}^j$.

Price Dispersion. In addition to the aggregate price index, we also need to track price dispersion. We would like a measure price dispersion that relates aggregate sectoral labor demand n_t^j to aggregate sectoral output y_t^j. Aggregate sectoral labor demand satisfies:

$$n_t^j = \int_i n_{it}^j = \int_i \left(\frac{y_{it}^j}{z_t^j}\right)^{\frac{1}{1-\alpha}}$$

Using the intermediate good demand function, we can write this as:

$$n_t^j = \left(\frac{y_t^j}{z_t^j}\right)^{\frac{1}{1-\alpha}} \left[\int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha}}\right]$$

This allows us to define a notion of price dispersion

$$\Delta_t^j = \int_i \left(\frac{p_{it}^j}{p_t^j}\right)^{-\frac{\epsilon}{1-\alpha}}$$

which satisfies:

$$y_t^j = z_t^j \left(\frac{n_t^j}{\Delta_t^j}\right)^{1-\alpha}$$

Dynamics of Price Dispersion. Price dispersion can be written as:

$$\Delta_t^j = \left(p_t^j\right)^{\frac{\epsilon}{1-\alpha}} \cdot \int_i \left(p_{it}^j\right)^{-\frac{\epsilon}{1-\alpha}}$$

Price dispersion evolves over time according to:

$$\Delta_t^j = \left(\Pi_t^j\right)^{\frac{\epsilon}{1-\alpha}} \left[\phi_j \left(\Delta_{t-1}^j\right) + (1-\phi_j)\left(\Pi_t^{j*}\right)^{-\frac{\epsilon}{1-\alpha}}\right]$$

D Omitted Proofs

Proof of Proposition 2. The flexible price equilibrium is the unique Pareto Optimum of the economy. Pareto Optimality requires $\tau_t^j = 0$, i.e. $p_t^j = w_t / f_{n_t}^j$. This implies that for every i, j , the relative price between sectors satisfies:

$$\frac{p_t^j}{p_t^i} = \frac{f_{n_t}^j}{f_{n_t}^i}$$

Since $f_{n_t}^j / f_{n_t}^i$ is pinned down by production, this implies that the relative price p_t^j / p_t^i is also pinned down. Due to the normalization $p_t^1 = 1$, this requires that $p_t^j = p_t^{j,flex}$ for all j. This is only feasible if $\bar{p}^j = p_t^{j,flex}$.

If there is linear production in each sector, then Pareto Optimality requires $p_t^j = z_t^j / z_t^i$. Since we start out at a Pareto Optimum, we know this holds for $\bar{p}^j$. Thus, in the absence of idiosyncratic sectoral demand shocks, this expression continues to hold. □

Proof of Proposition 1. The first-order condition of the optimal policy problem is:

$$\sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta \theta_{t+1}}{\theta_t} V_{c_t^j} \right) \frac{dc_t^j}{dR} = 0$$

The envelope conditions are:

$$V_{c_t^j} = \left(1 - \delta^j\right) \frac{v_{n_t}}{f_{n_t}^j}$$

Combining these we obtain:

$$\sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta \theta_{t+1}}{\theta_t} \left(1 - \delta^j\right) \frac{v_{n_{t+1}}}{f_{n_{t+1}}^j} \right) \frac{dc_t^j}{dR} = 0$$

Next we use that in the next period, since prices are flexible, we have:

$$w_{t+1} = \frac{v_{n_{t+1}}}{u_{c_{t+1}^1}} = p_{t+1}^j f_{n_{t+1}}^j$$

and therefore $\frac{v_{n_{t+1}}}{f_{n_{t+1}}^j} = p_{t+1}^j u_{c_{t+1}^1}$. This may not hold in period t, where instead we have:

$$\frac{v_{n_t}}{f_{n_t}^j} = \left(1 - \tau_t^j\right) p_t^j u_{c_t^1}$$

where τ_t^j is the labor wedge. Therefore optimal policy becomes (after dividing through by

$u_{c_t^1}$):

$$\sum_j \left(\frac{u_{c_t^j}}{u_{c_t^1}} + \frac{\beta\theta_{t+1}}{\theta_t} \left(1-\delta^j\right) p_{t+1}^j \frac{u_{c_{t+1}^1}}{u_{c_t^1}} - \left(1-\tau_t^j\right) p_t^j \right) \frac{dc_t^j}{dR} = 0$$

Now we substitute in the sector j asset pricing equation to obtain:

$$\sum_j \tau_t^j p_t^j \frac{dc_t^j}{dR} = 0$$

We now write this in terms of interest elasticities of demand and GDP shares. First since $y_t^j = c_t^j - \left(1-\delta^j\right) c_{t-1}^j$, it follows that $\frac{dy_t^j}{dR} = \frac{dc_t^j}{dR}$. Then we multiply and divide each term of the sum by y_t^j to put things in terms of production, multiply the entire expression through by $-R$, and then divide through by GDP, which is $y_t = \sum_j p_t^j y_t^j$. Then the expression can be written as:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = 0$$

where $\epsilon_R^j = -\frac{dy_t^j}{dR_{t+1}} \frac{R_{t+1}}{y_t^j}$, $\gamma_t^j = \frac{p_t^j y_t^j}{\sum_j p_t^j y_t^j}$, and $\tau_t^j = 1 - \frac{w_t}{p_t^j f_{n_{t+1}}^j}$. □

Proof of Proposition 3. Let ϵ and τ be random variables which take on values (ϵ^j, τ^j) in state j, which occurs with probability γ^j. Then $\epsilon^y = E[\epsilon]$, $\tau^y = E[\tau]$, and $\sum_j \gamma^j \left(\epsilon_R^j - \epsilon_R^y\right)\left(\tau^j - \tau^y\right) = \text{Cov}(\epsilon, \tau) = E[\epsilon\tau] - E[\epsilon]E[\tau]$. The static policy rule is then $E[\epsilon\tau] = \sum_j \gamma^j \epsilon_R^j \tau^j = 0$, and therefore under optimal policy we have $E[\epsilon]E[\tau] + \text{Cov}(\epsilon,\tau) = 0$. Dividing through by ϵ_R^y then yields the result. □

Proof of Proposition 4. The optimality expression is just as in the case with one-period fixed prices:

$$\sum_j \left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t} \left(1-\delta^j\right) \frac{v_{n_{t+1}}}{f_{n_{t+1}}^j} \right) \frac{dc_t^j}{dR} = 0$$

But now it may be the case that $\tau_{t+1}^j \neq 0$. Therefore we must use the expression:

$$\frac{v_{n_t}}{f_{n_t}^j} = \left(1-\tau_t^j\right) p_t^j u_{c_t^1}$$

in both period t and $t+1$. Using this, the expression above becomes:

$$\sum_j \left(\frac{u_{c_t^j}}{u_{c_t^1}} + \left(\frac{1-\delta^j}{R_{t+1}}\right) \left(1-\tau_{t+1}^j\right) p_{t+1}^j - \left(1-\tau_t^j\right) p_t^j \right) \frac{dc_t^j}{dR} = 0$$

Now we use the expression for p_t^j, together with the fact that $p_t^j = p_{t+1}^j = \bar{p}^j$, to obtain:

$$\sum_j \bar{p}^j \left(\tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j \right) \frac{dc_t^j}{dR} = 0$$

Now, as before, we note that $\frac{dy_t^j}{dR} = \frac{dc_t^j}{dR}$, we multiply and divide each term of the sum by y_t^j, multiply the entire expression by $-R$, and then divide through by GDP $y_t = \sum_j p_t^j y_t^j$, to obtain:

$$\sum_j \epsilon_R^j \gamma_t^j \tau_t^j = \sum_j \epsilon_R^j \gamma_t^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

□

Proof of Lemma 1. We show this by backward induction. First consider the expressions in period $T = N-1$, i.e. the last period with fixed prices. Here the expressions as:

$$\frac{u_{c_T^j}\left(\vec{c}_T\right)}{u_{c_T^1}\left(\vec{c}_T\right)} = \bar{p}^j - \frac{1-\delta^j}{R_{T+1}} p_{T+1}^j\left(\vec{c}_T\right)$$
$$\frac{u_{c_T^1}\left(\vec{c}_T\right)}{u_{c_{T+1}^1}\left(\vec{c}_T\right)} = \frac{\beta\theta_{T+1}}{\theta_T} R_{T+1}$$

where current marginal utility is a function of current consumption only from the assumption of time-separability of utility, and where future prices and consumption depend only on current consumption because these are defined by the flexible price equilibrium for given initial state, which here is just $\vec{c}_T$. Note that we have N_j equations in N_j unknowns, given entirely in terms of $(\vec{c}_T, R_{T+1}, \bar{p}^j)$. Thus these expressions implicitly define current demand as a function of current fixed prices and the current interest rate only:

$$\vec{c}_T\left(R_{T+1}\bar{p}^j\right)$$

Now we show by induction that demand equations for earlier periods depend only on future interest rates. Suppose this is true for all future periods up to period T. Then in period t we have:

$$\frac{u_{c_t^j}\left(\vec{c}_t\right)}{u_{c_t^1}\left(\vec{c}_t\right)} = \bar{p}^j \left(1 - \frac{1-\delta^j}{R_{t+1}}\right)$$
$$\frac{u_{c_t^1}\left(\vec{c}_t\right)}{u_{c_{t+1}^1}\left(\left\{R_s\right\}_{s\geq t+2}\right)} = \frac{\beta\theta_{t+1}}{\theta_t} R_{t+1}$$

These are again N_j equations in N_j unknowns, only in terms of $\left(\vec{c}_t, \vec{\bar{p}}, \left\{R_s\right\}_{s\geq t+1}\right)$. Therefore these expressions implicitly define $\vec{c}_t\left(\vec{p}, \{R_s\}_{s\geq t+1}\right)$. □

Proof of Proposition 5. Consider the choice of R_k for $k \in \{1, \dots, N\}$. The optimality condition is:

$$\sum_j \left[\sum_{t=0}^{N-2} \frac{\beta^t \theta_t}{\theta_0}\left(\frac{\partial U_t}{\partial c_t^j} + \frac{\beta\theta_{t+1}}{\theta_t}\frac{\partial U_{t+1}}{\partial c_t^j}\right)\frac{dc_t^j}{dR_k}\right] = 0$$

$$+\sum_j \left[\frac{\beta^{N-1}\theta_{N-1}}{\theta_0}\left(\frac{\partial U_{N-1}}{\partial c_{N-1}^j} + \frac{\beta\theta_N}{\theta_{N-1}}\frac{dV_N}{dc_{N-1}^j}\right)\frac{dc_{N-1}^j}{R_k}\right] = 0$$

Now we use the fact that:

$$U_{c_t^j} = u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} = u_{c_t^1}\left[\frac{u_{c_t^j}}{u_{c_t^1}} - \left(1-\tau_t^j\right)p_t^j\right]$$
$$U_{c_{t-1}^j} = \left(1-\tau_t^j\right)\left(1-\delta^j\right)u_{c_t^1}p_t^j$$

Then the optimality condition becomes:

$$\sum_j \left[\sum_{t=0}^{k-1} \beta^t \lambda_t \bar{p}^j \tau_t^j \left(1 - \left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\tau_{t+1}^j}{\tau_t^j}\right)\frac{dc_t^j}{dR_k}\right] = 0$$

Note that this holds for every $R_k \in \{R_1, \dots, R_N\}$. We can then write this as:

$$\sum_{t=0}^{k-1} \beta^t \lambda_t y_t \left[\sum_j \left(\gamma_t^j \tau_t^j - \gamma_t^j \tau_{t+1}^j\left(\frac{1-\delta^j}{R_{t+1}}\right)\right)\epsilon_{R_k}^{y_t^j}\right] = 0$$

□

Proof of Lemma 2. Demand for c_t^j for $j \neq 1$ is defined by the set of equations:

$$u_{c_t^j} = \left(1 - \frac{1-\delta^j}{R_{t+1}}\right)\bar{p}^j u_{c_t^1}$$
$$u_{c_t^1} = \frac{\beta\theta_{t+1}}{\theta_t}R_{t+1}u_{c_{t+1}^1}$$

Iterating the Euler equation forward in time, we obtain:

$$u_{c_t^1} = \beta^{N-t}\frac{\theta_N}{\theta_t}\left(\prod_{s=t+1}^{N} R_s\right)u_{c_N^1}$$

Substituting this into the system of equations above yields:

$$
\begin{aligned}
u_{c_t^j} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j u_{c_t^1} \\
u_{c_t^1} &= \beta^{N-t} \frac{\theta_N}{\theta_t} \left(\prod_{s=t+1}^{N} R_s \right) u_{c_N^1}
\end{aligned}
$$

Since there are N_j of these equations and N_j unknowns, this set of equations determines $\vec{c}_t \left(\{R_s\}_{s=t+1}^{N}, \bar{p}^j, \theta_t, \theta_N, u_{c_N^1} \right)$. Now consider R_k for $k \in [t+2, N]$. By Lemma 1, $u_{c_N^1}$ is not a function of R_k. Then if we differentiate the system of equations above by R_k, we obtain:

$$
\begin{aligned}
\sum_i u_{c_t^j c_t^i} \frac{dc_t^i}{dR_k} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j \left(\sum_i u_{c_t^1 c_t^i} \frac{dc_t^i}{dR_k} \right) \\
\sum_i u_{c_t^1 c_t^i} \frac{dc_t^i}{dR_k} &= \frac{u_{c_t^1}}{R_k}
\end{aligned}
$$

We can write this in terms of consumption demand interest elasticities as:

$$
\begin{aligned}
\sum_i u_{c_t^j c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} &= \left(1 - \frac{1-\delta^j}{R_{t+1}}\right) \bar{p}^j \left(\sum_i u_{c_t^1 c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} \right) \\
\sum_i u_{c_t^1 c_t^i} c_t^i \epsilon_{R_k}^{c_t^i} &= -u_{c_t^1}
\end{aligned}
$$

This yields N_j equations in N_j unknowns, namely the interest elasticities $\epsilon_{R_k}^{c_t^i}$. But note that these expressions are the same for any $k \in [t+2, N]$, and thus the elasticities are the same. This proves the proposition. □

Proof of Proposition 6. We can write the optimality expressions as:

$$
\beta^{k-1} \lambda_{k-1} y_{k-1} \left[\sum_j \left(\gamma_{k-1}^j \tau_{k-1}^j - \gamma_{k-1}^j \tau_k^j \left(\frac{1-\delta^j}{R_{k1}} \right) \right) \epsilon_{R_k}^{y_{k-1}^j} \right] = - \sum_{t=0}^{k-2} \beta^t \lambda_t y_t \left[\sum_j \left(\gamma_t^j \tau_t^j - \gamma_t^j \tau_{t+1}^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \right) \epsilon
$$

Take the difference:

$$
\beta^{k-1} \lambda_{k-1} y_{k-1} \left[\sum_j \left(\gamma_{k-1}^j \tau_{k-1}^j - \gamma_{k-1}^j \tau_k^j \left(\frac{1-\delta^j}{R_{k1}} \right) \right) \epsilon_{R_k}^{y_{k-1}^j} \right] = \sum_{t=0}^{k-2} \beta^t \lambda_t y_t \left[\sum_j \left(\gamma_t^j \tau_t^j - \gamma_t^j \tau_{t+1}^j \left(\frac{1-\delta^j}{R_{t+1}} \right) \right) \left(\epsilon
$$

The right-hand terms cancel for $t < k-2$. Then we obain:

$$\sum_j \gamma^j_{k-1}\left(\tau^j_{k-1} - \tau^j_k\left(\frac{1-\delta^j}{R_{k1}}\right)\right)\epsilon^{y^j_{k-1}}_{R_k} = \frac{R_{k-1}y_{k-2}}{y_{k-1}}\left[\sum_j \gamma^j_{k-2}\left(\tau^j_{k-2} - \tau^j_{k-1}\left(\frac{1-\delta^j}{R_{k-1}}\right)\right)\left(\epsilon^{y^j_{k-2}}_{R_{k-1}} - \epsilon^{y^j_{k-2}}_{R_k}\right)\right]$$

□

Proof of Proposition 7. The demand equations can be written:

$$u_{c^j_t} = \left(1 - \frac{1-\delta^j}{R_1}\right)\bar{p}^j u_{c^1_t}$$
$$u_{c^1_t} = \beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$

where the second equation is the Euler equation iterated forward an extra period. Combining these, we obtain:

$$u_{c^j_t} = \left(1 - \frac{1-\delta^j}{R_1}\right)\bar{p}^j\beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$
$$u_{c^1_t} = \beta^2\frac{\theta_{t+2}}{\theta_t}R_{t+1}R_{t+2}u_{c^1_{t+2}}$$

Since (by Lemma 1) c^j_{t+2} does not depend on R_{t+1} or R_{t+2}, we can immediately compute the following:

$$u_{c^j_t c^j_t}\frac{dc^j_t}{dR_{t+2}} = \frac{u_{c^j_t}}{R_{t+2}}$$
$$u_{c^j_t c^j_t}\frac{dc^j_0}{dR_{t+1}} = \frac{u_{c^j_t}}{r_{t+1}+\delta^j}$$

We can express this as:

$$\sigma^j_t\epsilon^{y^j_t}_{R_{t+2}}\frac{y^j_t}{c^j_t} = 1$$
$$\sigma^j_t\epsilon^{y^j_0}_{R_{t+1}}\frac{y^j_t}{c^j_t} = \frac{1+r_{t+1}}{r_{t+1}+\delta^j}$$

where $\sigma^j_t = -\frac{u_{c^j_t c^j_t}c^j_t}{u_{c^j_t}}$. Or in other words:

$$\epsilon^{y^j_t}_{R_{t+2}} = \left(\frac{r_{t+1}+\delta^j}{1+r_{t+1}}\right)\epsilon^{y^j_t}_{R_{t+1}}$$

Taking the difference, this implies:

$$\epsilon^{y^j_t}_{R_{t+1}} - \epsilon^{y^j_t}_{R_{t+2}} = \left(\frac{1-\delta^j}{R_{t+1}}\right)\epsilon^{y^j_t}_{R_{t+1}}$$

Plugging this into the optimal policy expression found in 5 yields the last expression. □

Proof of Proposition 10. As before, labor can be expressed as a function of demand as:

$$n_t\left(\vec{c}_t\right) = \sum_j \left(f^j_t\right)^{-1}\left(c^j_t - \left(1-\delta^j\right)c^j_{t-1}\right)$$

Then the optimal policy problem is to choose R_{t+1} to maximize:

$$V_t\left(\vec{c}_{t-1}\right) = \max_R \left\{u\left(\vec{c}_t\left(R\right)\right) - v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right) + \mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}V_{t+1}\left(\vec{c}_t\left(R\right)\right)\right]\right\}$$

The optimality expression is:

$$\sum_j \left(u_{c^j_t} + \mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}V_{c^j_t}\right] - \frac{v_{n_t}}{f^j_{n_t}}\right)\frac{dc^j_t}{dR} = 0$$

From the envelope condition, we have:

$$V_{c^j_{t-1}} = \left(1-\delta^j\right)\frac{v_{n_t}}{f^j_{n_t}}$$

But since the next period has flexible prices, this implies:

$$V_{c^j_t} = \left(1-\delta^j\right)p^j_{t+1}\lambda_{t+1}$$

Substituting this into the expression above, we obtain:

$$\sum_j \left(u_{c^j_t} + \mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}\left(1-\delta^j\right)p^j_{t+1}\lambda_{t+1}\right] - \frac{v_{n_t}}{f^j_{n_t}}\right)\frac{dc^j_t}{dR} = 0$$

This we may write this as:

$$\sum_j \lambda_t p^j_t\left(1 - \frac{w_t}{p^j_t f^j_{n_t}}\right)\frac{dc^j_t}{dR} = 0$$

which, as before, we may write as:

$$\sum_j \epsilon^j_R \gamma^j_t \tau^j_t = 0$$

□

Proof of Proposition 11. We again write the optimal policy problem as:

$$V_t\left(\vec{c}_{t-1}\right)=\max_R\left\{u\left(\vec{c}_t\left(R\right)\right)-v\left(n_t\left(\vec{c}_t\left(R\right)\right)\right)+\mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}V_{t+1}\left(\vec{c}_t\left(R\right)\right)\right]\right\}$$

The optimality expression is again:

$$\sum_j\left(u_{c_t^j}+\mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}V_{c_t^j}\right]-\frac{v_{n_t}}{f_{n_t}^j}\right)\frac{dc_t^j}{dR}=0$$

But now it may be the case that $\tau_{t+1}^j\neq 0$. Thus we must use the expression:

$$V_{c_t^j}=\left(1-\delta^j\right)\frac{v_{n_{t+1}}}{f_{n_{t+1}}^j}=\left(1-\delta^j\right)\left(1-\tau_{t+1}^j\right)p_{t+1}^j\lambda_{t+1}$$

Using this, the expression above becomes:

$$\sum_j\left(u_{c_t^j}+\left(1-\delta^j\right)\mathbb{E}_t\left[\frac{\beta\theta_{t+1}}{\theta_t}\left(1-\tau_{t+1}^j\right)p_{t+1}^j\lambda_{t+1}\right]-\frac{v_{n_t}}{f_{n_t}^j}\right)\frac{dc_t^j}{dR}=0$$

Now we use the expression for p_t^j, together with the fact that $p_t^j=p_{t+1}^j=\bar{p}^j$, and the definition of R_{t+1} to obtain:

$$\sum_j\bar{p}^j\left(\tau_t^j-\left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}\right)\frac{dc_t^j}{dR}=0$$

Now, as before, we note that $\frac{dy_t^j}{dR}=\frac{dc_t^j}{dR}$, we multiply and divide each term of the sum by y_t^j, multiply the entire expression by $-R$, and then divide through by GDP $y_t=\sum_j p_t^j y_t^j$, to obtain:

$$\sum_j\epsilon_R^j\gamma_t^j\tau_t^j=\sum_j\epsilon_R^j\gamma_t^j\left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\mathbb{E}_t\left[\theta_{t+1}\tau_{t+1}^j\lambda_{t+1}\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}$$

□

Proof of Proposition 12. The problem of the monetary authority is:

$$\max_{\{R_{t+1}\}_{t=0}^{N-1}}\sum_{t=0}^{N-1}\sum_{s^t}q\left(s^t\right)\beta^t\theta_t\left[u\left(\vec{c}_t\right)-v\left(n_t\left(\vec{c}_t,\vec{c}_{t-1}\right)\right)\right]+\beta^T\sum_{S^T}q\left(s^T\right)\theta_T V\left(\vec{c}_{T-1}\right)$$

As before, past choices of interest rates do not affect future equilibrium variables. Thus the optimality condition for the choice of R_{k+1} is:

$$\sum_{t=0}^{k}\sum_{s^t} q\left(s^t\right)\beta^t\theta_t\sum_j\left[\left(u_{c_t^j}-\frac{v_{n_t}}{f_{n_t^j}}\right)\left(c_t^j\right)_{R_{k+1}}\right]+\sum_{t=1}^{k+1}\sum_{s^t} q\left(s^t\right)\beta^t\theta_t\sum_j\left(1-\delta^j\right)\frac{v_{n_t}}{f_{n_t^j}}\left(c_{t-1}^j\right)_{R_{k+1}}=0$$

We reindex to write these as:

$$\sum_{t=0}^{k}\sum_{s^t} q\left(s^t\right)\beta^t\theta_t\sum_j\left[u_{c_t^j}-\frac{v_{n_t}}{f_{n_t^j}}+\beta\left(1-\delta^j\right)\sum_{s^{t+1}\subset s^t}\frac{q\left(s^{t+1}\right)}{q\left(s^t\right)}\frac{\theta_{t+1}}{\theta_t}\frac{v_{n_{t+1}}}{f_{n_{t+1}^j}}\right]\left(c_t^j\right)_{R_{k+1}}=0$$

Using $v_{n_t}/f_{n_t^j}=\left(1-\tau_t^j\right)\lambda_t p_t^j$ and $u_{c_t^j}=p_t^j\lambda_t-\beta\left(1-\delta^j\right)\mathbb{E}_t\frac{\theta_{t+1}}{\theta_t}\lambda_{t+1}p_{t+1}^j$, we can write this as:

$$\sum_{t=0}^{k}\sum_{s^t} q\left(s^t\right)\beta^t\theta_t\sum_j\left[\tau_t^j-\left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\sum_{s^{t+1}\subset s^t} q\left(s^{t+1}\right)\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j}{\sum_{s^{t+1}\subset s^t} q\left(s^{t+1}\right)\theta_{t+1}\lambda_{t+1}}\right]\lambda_t p^j\left(c_t^j\right)_{R_{k+1}}=0$$

or just:

$$\mathbb{E}_0\sum_{t=0}^{k}\beta^t\theta_t\lambda_t y_t\sum_j\gamma_t^j\epsilon_{R_{k+1}}^{y_t^j}\chi_t^j=0$$

where

$$\chi_t^j=\tau_t^j-\left(\frac{1-\delta^j}{R_{t+1}}\right)\frac{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_t\left[\theta_{t+1}\lambda_{t+1}\right]}$$

is the durable overhang-augmented labor wedge. □

Proof of Lemma 3 . As in the certainty case, we can iterate the Euler equation forward to show that the effect of the interest more than 1 period ahead is the same. First observe that the good j demand equation at time $t<N-1$ can be written as:

$$u_{c_t^j}=\left(1-\frac{1-\delta^j}{R_{t+1}}\right)p^j\lambda_t$$

Since p^j is fixed, the only moving parts here are R_{t+1} and λ_t. By iterating the Euler equation forward, we obtain:

$$\theta_t\lambda_t=\beta^{N-t}\left(\prod_{k=t}^{N-1}R_{k+1}\right)\mathbb{E}_t\left[\theta_N\lambda_N\right]$$

Note that R_{k+1} for $k\in[t+1,N-1]$ has exactly the same effect on demand. □

Proof of Proposition 13. Start with the objective function

$$\sum_{t=0}^{N-1}\sum_{S^t} q\left(s^t\right)\beta^t\theta_t\left[u\left(\vec{c}_t\right)-v\left(n_t\left(\vec{c}_t,\vec{c}_{t-1}\right)\right)\right]+\beta^T\sum_{S^T} q\left(s^T\right)\theta_T V\left(\vec{c}_{T-1}\right)$$

The first-order condition with respect to $R_{k+1}\left(s_*^k\right)$ satisfies:

$$\sum_{t=0}^{k} q\left(s_*^t\right)\beta^t\theta_t\sum_j\left(u_{c_t^j}-\frac{v_{n_t}}{f_{n_t}^j}\right)\frac{dc_t^j}{dR_{k+1}\left(s_*^k\right)} \\ +\sum_{t=0}^{k}\sum_{s^{t+1}\subset s_*^t} q\left(s^{t+1}\right)\beta^{t+1}\theta_{t+1}\sum_j\left[\left(1-\delta^j\right)\frac{v_{n_{t+1}}}{f_{n_{t+1}}^j}\frac{dc_t^j}{dR_{k+1}\left(s_*^k\right)}\right]=0$$

where the states s_*^t are taken to lie on the path defined by s_*^k, that is $s_*^k\subset s_*^t$. Note that since the choice of $R_{k+1}\left(s_*^k\right)$ may depend on the entire history of past states, its effect on c_t^j only occurs in the state s_*^t, i.e. the state that yields directly to s_*^k. However, this increase in $c_t^j\left(s_*^t\right)$ will lead to an increase in the initial stock of goods $c_t^j\left(s^{t+1}\right)$ in all state s^{t+1} reachable from s_*^t, including ones not leading to s_*^k. Now we use the fact that $u_{c_t^j}=\bar{p}^j\lambda_t\left(1-\frac{1-\delta^j}{R_{t+1}}\right)$, $\frac{v_n}{f_n^j}=\left(1-\tau_t^j\right)\bar{p}^j\lambda_t$, and

$$\frac{1}{R_{t+1}\left(s_*^t\right)}=\mathbb{E}_{t*}\left[\beta\frac{\theta_{t+1}}{\theta_t}\frac{\lambda_{t+1}}{\lambda_t}\right]=\sum_{s^{t+1}\subset s_*^t}\left[\frac{q\left(s^{t+1}\right)}{q\left(s_*^t\right)}\beta\frac{\theta_{t+1}}{\theta_t}\lambda_{t+1}\right]$$

to obtain:

$$\sum_{t=0}^{k} q\left(s_*^t\right)\beta^t\theta_t\lambda_t y_t\left\{\sum_j\epsilon^{y_t^j}_{R_{k+1}\left(s_*^k\right)}\gamma_t^j\left(\tau_t^j-\left(\frac{1-\delta^j}{R_{t+1}\left(s_*^t\right)}\right)\frac{\mathbb{E}_{t*}\left[\theta_{t+1}\lambda_{t+1}\tau_{t+1}^j\right]}{\mathbb{E}_{t*}\left[\lambda_{t+1}\theta_{t+1}\right]}\right)\right\}=0$$

□

Proof of Proposition 8. Consider the N-period fixed price case without commitment. We write the problem of the policymaker as:

$$V_t\left(\vec{c}_{t-1}\right)=\max_{R_{t+1}}\left\{u\left(\vec{c}_t,\vec{c}_{t-1}\right)-v\left(n_t\left(\vec{c}_t,\vec{c}_{t-1}\right)\right)+\frac{\beta\theta_{t+1}}{\theta_t}V_{t+1}\left(\vec{c}_t\right)\right\}=\max_{R_{t+1}}\left\{U_t\right\}$$

where V_{t+1} is defined recursively, with V_{t+N} being the flexible price value function. We again have:

$$U_{c_t^j}=u_{c_t^j}-\frac{v_{n_t}}{f_{n_t}^j}+\frac{\beta\theta_{t+1}}{\theta_t}V_{c_t^j}$$

The envelope condition is:

$$V_{c_{t-1}^j} = (u_t)_{c_{t-1}^j} + \left(1-\delta^j\right)\frac{v_{n_t}}{f_{n_t}^j} = (u_t)_{c_{t-1}^j} + \left(1-\delta^j\right)p_t^j\lambda_t\left(1-\tau_t^j\right)$$

Thus we obtain:

$$\begin{aligned} U_{c_t^j} &= u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \frac{\beta\theta_{t+1}}{\theta_t}\left[(u_{t+1})_{c_t^j} + \left(1-\delta^j\right)p_{t+1}^j\lambda_{t+1}\left(1-\tau_{t+1}^j\right)\right] \\ &= p_t^j\lambda_t\tau_t^j - \lambda_t\left(\frac{1-\delta^j}{R_{t+1}}\right)p_{t+1}^j\tau_{t+1}^j \end{aligned}$$

Now the optimality condition is just as before (after observing that $p_{t+1}^j = p_t^j$):

$$\sum_j \epsilon_R^j\gamma_t^j\left(\tau_t^j - \left(\frac{1-\delta^j}{R_{t+1}}\right)\tau_{t+1}^j\right) = 0$$

which can also be written:

$$\sum_j \epsilon_R^j\gamma_t^j\tau_t^j = \sum_j \epsilon_R^j\gamma_t^j\left(\frac{1-\delta^j}{R_{t+1}}\right)\tau_{t+1}^j$$

The expression for $N=1$ is implied, after we note that in this case $\tau_{t+1}^j = 0$. □

Proof of Proposition 9. With commitment the central bank chooses the path of interest rates $\{R_k\}_{k=1}^N$ to maximize objective function:

$$\sum_{t=0}^{N-1}\beta^t\theta_t\left[u\left(\vec{c}_t,\vec{c}_{t-1}\right) - v\left(n_t\left(\vec{c}_t,\vec{c}_{t-1}\right)\right)\right] + \beta^N\theta_N V_N\left(\vec{c}_{N-1}\right)$$

where $V_N\left(\vec{c}_{N-1}\right)$ is the flexible price value function entering period N. The optimal choice of R_k satisfies:

$$\sum_{t=0}^{N-1}\beta^t\theta_t\sum_j\left(u_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j}\right)\left(c_t^j\right)_{R_k} + \sum_{t=1}^{N}\beta^t\theta_t\sum_j\left[\left((u_t)_{c_{t-1}^j} + \left(1-\delta^j\right)\frac{v_{n_t}}{f_{n_t}^j}\right)\left(c_{t-1}^j\right)_{R_k}\right] = 0$$

Adjusting the time indices yields:

$$\sum_{t=0}^{N-1}\beta^t\theta_t\sum_j\left\{\left(u_{c_t^j} + \beta\frac{\theta_{t+1}}{\theta_t}(u_{t+1})_{c_t^j} - \frac{v_{n_t}}{f_{n_t}^j} + \beta\frac{\theta_{t+1}}{\theta_t}\left(1-\delta^j\right)\frac{v_{n_{t+1}}}{f_{n_{t+1}}^j}\right)\left(c_t^j\right)_{R_k}\right\} = 0$$

Using $v_{n_t}/f_{n_t}^j = \lambda_t p_t^j\left(1-\tau_t^j\right)$ and $u_{c_t^j} + \beta\frac{\theta_{t+1}}{\theta_t}(u_{t+1})_{c_t^j} = p_t^j\lambda_t - \frac{1-\delta^j}{R_{t+1}}p_{t+1}^j\lambda_t$, we can write this as:

$$\sum_{t=0}^{N-1} \beta^t \theta_t \lambda_t y_t \sum_j \gamma_t^j \epsilon_{R_k}^{y_t^j} \chi_t^j = 0$$

where

$$\chi_t^j = \tau_t^j - \left(\frac{1 - \delta^j}{R_{t+1}} \right) \tau_{t+1}^j$$

is the overhang-augmented labor wedge. □