Household Debt, Unemployment, and Slow Recoveries*

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January 2016

Abstract

I study the dynamics of an economy with endogenous unemployment risk in a liquidity trap. Expectations of high unemployment increase precautionary saving and worsen credit conditions, reducing demand. When the interest rate cannot fall in response, pessimistic expectations are self-fulfilling and multiple equilibria exist. Even under optimistic expectations, endogenous unemployment risk substantially amplifies employment losses in a deleveraging episode. I decompose demand losses into deleveraging and precautionary components, and find that precautionary effects account for most of the amplification. A concurrent temporary financial shock causes more initial unemployment but a faster recovery, and can reduce cumulative employment losses.

JEL Classification: E21, E44, E32, E24.

Keywords: Liquidity Trap, Multiple Equilibria, Precautionary Saving.

*I would like to thank Anton Korinek for his advice, encouragement, and many helpful comments. I would like to thank John Shea, Borağan Aruoba, John Haltiwanger, Luminita Stevens, and Sebnem Kalemli-Ozcan for helpful conversations. I would like to thank participants of the UMD Macro Workshop for helpful comments. First draft: November 2014.

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1 Introduction

In the wake of the 2007 – 2009 recession, the U.S. experienced a period of persistently high unemployment with short-term interest rates at the zero lower bound. Several studies have linked the severity of the recession and the slow recovery of hiring to weak consumer demand, possibly due to high levels of household debt.[1] In this paper I explore an amplification channel deriving from endogenous unemployment risk, which operates whenever monetary policy is constrained. This mechanism generates multiple equilibrium paths of recovery following any shock that produces a liquidity trap.[2]

When monetary policy is unconstrained, determinacy is achieved through the expectation that the central bank will achieve its target level of employment. However, when the real rate is bounded from below, as in a liquidity trap, pessimistic expectations about the pace of recovery can be self-fulfilling. Even under optimistic expectations, this channel substantially increases employment losses following a demand shock, such as a deleveraging episode. This channel may explain the magnitude and persistence of employment losses following the 2008 financial crisis.

The feedback channel I analyze relies on slow hiring weakening demand. This happens for two reasons. First, slow hiring implies longer spells of unemployment, which prompt greater precautionary saving by households. Second, high unemployment endogenously worsens financial constraints facing households, because longer expected spells of unemployment increase the incentives for households to default on their debt. When the economy is demand-constrained, as in a liquidity trap, the fall in desired spending causes employment to fall further.

To explore this mechanism, I develop a model in which households face idiosyncratic endogenous unemployment risk. Labor is indivisible, so that a fraction of households are unemployed at any time. Households face a time-varying job-finding rate, which is lower during periods of low employment. Households cannot insure against employment loss, and so employed households save and unemployed households dissave to smooth consumption over unemployment spells. Lenders impose a borrowing constraint on households, set to prevent default. As in a standard New Keynesian model, prices are sticky

[1]Mian and Sufi (2010), and Mian, Rao, and Sufi (2013) find that high household leverage contributed to the decline in employment and consumption in 2008. Dynan (2012) and Hall (2011) argue that high levels of household debt contributed to the slow recovery since 2008. Reinhart and Rogoff (2009), Hall (2010), and Jordà, Schularick, and Taylor (2013) find that financial crises are often preceded by increases in leverage and followed by slow recoveries, but for a contrary finding see Romer and Romer (2015).

[2]Indeterminacy for a path of the *nominal* interest rate is common in New Keynesian models. See, for example, Bernanke and Woodford (1997), Benhabib, Schmitt-Grohé, and Uribe (2001), Clarida, Galí, and Gertler (2000), and Cochrane (2011). However, standard models all have a unique equilibrium for a given path of the *real* interest rate under the assumption of convergence to the steady state.

and the levels of output and employment are determined by demand. Unlike in a standard model, the path of output in turn determines the rate of job-finding, which affects demand through the precautionary and credit channels described above.

I consider an economy in which some households have accumulated a high level of debt, requiring a period of deleveraging.[3] This leads to a period of reduced aggregate spending, as highly indebted households seek to reduce their debts. As the initial asset distribution does not affect productive capacity, under flexible prices the interest rate would fall to raise desired consumption and maintain the steady state level of output. If instead prices are sticky, the central bank will try to replicate the flexible-price equilibrium by lowering the interest rate. However, if the economy is in a liquidity trap, the central bank will not be able to lower the interest rate sufficiently to offset the reduction in demand due to the high levels of household debt, and employment will fall instead.[4]

My first main result is that there are multiple equilibria possible when the economy falls into a liquidity trap and the central bank lacks commitment. A lower path of employment implies a lower job-finding rate, which raises precautionary saving and tightens borrowing constraints facing households, reducing demand. Thus the expectation of a slower recovery is self-fulfilling. Multiple equilibria exist whenever demand is sufficiently sensitive to the job-finding rate. I discuss this condition in section 3.3, and show that it holds for my model.

My second main result is that, even under optimistic expectations, a deleveraging episode produces a lengthy period of depressed employment, and endogenous unemployment risk significantly *amplifies* the fall of employment during the deleveraging episode.[5] I calibrate the baseline deleveraging episode to produce a 5 pct. point initial fall in the employment rate, the same as the increase in the unemployment rate in the U.S. from the first quarter of 2007 to the third quarter of 2009. This produces a slow recovery in the labor market, with employment still depressed by 0.7 pct. point after 20 quarters, and full recovery not achieved until 60 quarters. Since employment recovers slowly, the hiring rate remains depressed for some time as well. This period of slow hiring reduces demand via the precautionary and credit channels discussed above. This feedback amplifies employment losses in the deleveraging episode, producing a larger

[3]Technically, I consider an initial asset distribution with greater dispersion than steady state. The baseline experiment simply takes this asset distribution as given, but Appendix A.1 shows that this experiment is equivalent to a permanent tightening of lending standards.

[4]This result is similar to Eggertsson and Woodford (2003), Werning (2011), and Eggertsson and Krugman (2012).

[5]By optimistic expectations, I mean the highest path of employment consistent with the central bank's no commitment policy rule. Better equilibria are possible if the central bank can commit to allowing higher employment in the future.

fall in employment than would otherwise occur.

To determine the source of reduced demand during the deleveraging episode, I decompose the aggregate Euler equation into terms corresponding to interest rate effects, precautionary saving, and forced deleveraging by constrained households. This decomposition implies that 55% of the initial fall in employment, and 81% of the cumulative fall, is due to increased precautionary saving. This suggests that when analyzing deleveraging episodes it is important to consider precautionary effects throughout the asset distribution, and not just reductions in spending by constrained households.

To assess the magnitude of this amplification channel, I compute the response to a reduced form demand shock in a complete-markets version of the main model. I define the reduced form demand shock to be equivalent to the depressed demand due to the initial high debt asset distribution. Specifically, I compute the implied natural rate of interest in the full model, and then consider the effect of an exogenous reduction in the natural rate of interest in a complete markets model. I find that the initial fall in employment in the complete-markets benchmark is only 40% of the baseline model, implying that the amplification channel increases employment loss by a factor of 2.5. The magnitude of this amplification suggests it is important to account for endogenous variation of unemployment risk in incomplete market models, which most existing models neglect.[6]

Since the household deleveraging in 2008 took place in the context of a severe financial crisis, I also consider a deleveraging experiment in which household access to credit is temporarily restricted. Thus there are two exogenous deviations from steady state — a high initial level of debt that requires a period of deleveraging, and a temporary tightening of household credit. This results in a much greater initial fall in employment, but a much faster recovery, so that the *cumulative* employment loss in employment-quarters is somewhat *less* than in the baseline deleveraging episode. The tighter borrowing constraint facing households due to the temporary credit shock causes highly indebted households to reduce their debt holdings, a period of rapid deleveraging that reduces demand and causes employment to fall more in the short run. However, this initial period of deleveraging reduces the total debt burden, so that less deleveraging remains to be done after the credit shock has dissipated. Intuitively, the temporary shock directly to household credit access purges bad balance sheets, so that households are able to more rapidly increase spending as credit conditions recover. This faster recovery reduces feedback from future slow hiring, so that the cumulative employment losses are

[6]The standard approach in incomplete market models is to assume an exogenous idiosyncratic productivity process for households. Then the only endogenous variation in income risk is from changes in wages, but this variation is *countercyclical*, i.e. when wages fall during a slump, the variance of income risk facing households *falls*. For a survey of this literature, see Heathcote, Storesletten, and Violante (2009).

less despite the exogenous demand shock being much greater. This result hints at a *liquidationist* perspective — in some cases it is better to let a crisis run its course and liquidate excess accumulation of debt, laying the groundwork for a faster recovery.[7]

I also analyze how the presence of unemployment insurance affects the dynamics of recovery. I find that unemployment insurance reduces the initial fall in employment during a deleveraging episode by reducing the endogenous variance in income risk facing households. However, the reduction in income risk reduces the urgency to deleverage, producing a slower recovery and partially offsetting the positive employment effects of unemployment insurance.

I discuss alternative equilibria in section 6. I compute a path of recovery following the baseline deleveraging experiment under pessimistic expectations, and show that it results in a greater initial fall in output and a slower recovery. I also analyze a pure expectations shock, in which self-fulfilling expectations of a period of reduced hiring cause a recession. This can happen if households' loss of confidence is sufficiently large that they require a negative real interest rate to offset, causing the economy to fall into a liquidity trap. However, I find that such an event produces an initial increase in household debt, making it an unlikely explanation for the 2007 – 2009 recession.

The existence of multiple equilibria suggests that the central bank can improve matters by raising expectations. Several authors, starting with Krugman (1998), have pointed out that the central bank can reduce employment losses in a liquidity trap by promising to keep interest rates low after the recovery, a policy known as *forward guidance*. Future low rates generate a post-recovery boom, which raises demand in earlier periods due to consumption smoothing. In my formulation, a future boom also implies a higher path of *hiring*, which raises demand by reducing precautionary saving. However, as emphasized by all these authors, engineering such a boom requires the central bank to commit to following an ex post suboptimal policy once the economy exits the trap.

Literature. This paper is part of a rapidly growing literature on the dynamics of the economy in a liquidity trap. The early papers in this literature, Krugman (1998) and Eggertsson and Woodford (2003), were the first to consider the possibility of a binding zero lower bound on nominal interest rates in a modern context, and suggested the possibility of forward guidance in substituting for conventional monetary policy in these circumstances. Werning (2011) investigates optimal policy in this setting, and clarifies the dynamics of a liquidity trap in a simple and elegant model. Cochrane (2011) and Aruoba

[7] See discussion of liquidationist views in Rognlie, Shleifer, and Simsek (2014) and in Beaudry, Galizia, and Portier (2014), who analyze the liquidation of an excess accumulation of capital.

and Schorfheide (2013) discuss the problem of determinacy in the presence of a zero lower bound, and the possibility of deflationary traps, which is related to my finding of multiple equilibrium paths of recovery.

A number of papers have highlighted the role of household debt and deleveraging as the cause of low demand leading to a liquidity trap. Eggertsson and Krugman (2012) first explored this deleveraging channel in the context of a simple model with a zero lower bound. Hall (2011) develops a similar hypothesis, with particular reference to high levels of household debt producing a slow recovery of demand. Korinek and Simsek (2014) study the role of macroprudential regulation in mitigating this deleveraging channel. Guerrieri and Lorenzoni (2011) investigate the effect of precautionary saving behavior on the dynamics of deleveraging. The present paper contributes to this literature by studying endogenous time-varying unemployment risk in a liquidity trap, and studying the resulting amplification through precautionary saving and endogenous borrowing constraints.

A few papers have recently considered the interaction of precautionary savings and endogenous unemployment risk. Challe et al. (2014) and Ravn and Sterk (2013) combine these ingredients with sticky prices, and show that variations in precautionary saving over the business cycle amplify employment fluctuations. Caggese and Perez (2013) consider constraints facing both firms and households, and show that precautionary behavior by households and firms can interact to increase the volatility of employment. Beaudry, Galizia, and Portier (2014) study endogenous unemployment risk in a model with decentralized markets and search frictions. None of these papers analyze endogenous unemployment risk in a liquidity trap, and thus none of them feature multiple equilibria and the essential role for expectations found in the present paper. This paper also sheds additional light on the mechanism behind this amplification process, showing that it operates through a depressed job-finding rate, and providing a measure of amplification and a decomposition between precautionary and deleveraging components.

Several papers have empirically investigated the household debt / aggregate demand hypothesis of the 2007 – 2009 recession. Mian and Sufi (2010) and Mian, Rao, and Sufi (2013) use detailed county-level data to show that counties with high household leverage and large declines in house prices before the crisis had larger declines in employment and output during the crisis. Mian and Sufi (2014) shows that these differential employment declines are driven by the hiring decisions of nontradable good firms, suggesting that the mechanism operates through a demand channel. Dynan (2012) finds that households with high leverage saw larger declines in consumption in 2007 – 2009, despite a smaller decline in net worth, indicating the existince of a household credit channel rather than a wealth channel. Baker (2014) finds that households with higher levels of debt adjust their consumptiion more in response to changes in income, suggesting a higher marginal propensity to consume for highly indebted households.

Several authors have recently offered alternative models of aggregate demand channels. Kocherlakota (2012) analyzes a so-called incomplete labor market model, in which the real interest rate is set by the central bank and the labor supply condition may not hold. This formulation is similar to a New Keynesian model with fixed prices, except that the labor supply condition is dropped instead of labor demand. Chamley (2014) investigates the possibility of saving traps, in which self-fulfilling demand for savings may produce a low employment equilibrium, or slow convergence towards full employment. Michaillat (2012) argues that matching frictions are insufficient to explain unemployment and develops a model of job rationing during recessions. Michaillat and Saez (2013) develop a model of aggregate demand with matching frictions in both labor and goods markets, and show that tightness in one market affects tightness in the other, generating an aggregate demand channel for employment fluctuations. I view these approaches as complementary to the New Keynesian formulation of aggregate demand used in this paper.

Since this paper's primary mechanism operates through time-varying precautionary saving by households, empirical evidence of such saving is highly relevant. Carroll and Samwick (1997) and Carroll and Samwick (1998) find that households that face greater income uncertainty hold more wealth, and estimate that precautionary saving accounts for $39-46\%$ of household asset holdings. Parker and Preston (2005) find a significant and strongly countercyclical precautionary saving motive, that is similar in magnitude to the interest-rate motive. Carroll, Slacalek, and Sommer (2012) find that a significant portion of the consumption decline after the 2008 financial crisis was attributable to precautionary effects. McKay (2014) likewise finds that precautionary saving responses to increased household income risk explains about 2/3 of the drop in aggregate consumption during the Great Recession.

2 Households

The model is set in continuous time with a single non-storable consumption good. There are three types of agents: households, final-good firms, and intermediate-good firms. I first discuss the household problem, and then turn to the rest of the model.

There is a measure 1 of households, indexed by $i \in [0,1]$. Household i has expected lifetime utility

$$E_0\left[\int_0^\infty e^{-\rho t}u(c_i(t))dt\right]$$

where $u(\cdot)$ is a standard utility function, and ρ is the household subjective discount rate. I assume throughout that $u(c)$ exhibits constant relative risk aversion γ, i.e. $u(c)=\frac{c^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ and $u(c)=\log(c)$ for $\gamma = 1$.

2.1 Income Process

Households receive a flow of nonlabor income $e(t)$, which is identical across all households. $e(t)$ includes both a fixed endowment income flow $\bar{e}$, and profits from firms $\pi(t)$. Household i receives a flow of labor income $w(t)h_i(t)$, where $h_i(t) \in \{0,1\}$ is household i's employment status at time t, and $w(t)$ is the wage of employed workers. Labor is indivisible, so households are either employed or unemployed at each time t.

Since households suffer no disutility from labor, all households would like to work, but they are unable to do so because of indivisible labor and matching frictions in the labor market. Unemployed households receive job-finding shocks with probability $p(t)$, and employed households receive separation shocks with probability $s(t)$ per unit time.[8]

Household i enters period t with net assets $a_i(t)$, and can save or borrow at net real interest rate $r(t)$. Assuming no default, household assets evolve according to

$$\dot{a}_i(t) = r(t)a_i(t) + e(t) + w(t)h_i(t) - c_i(t) \tag{1}$$

where $\dot{a}_i(t)$ is household i's net saving.

Households face a time-varying borrowing constraint, which they take as exogenous. This borrowing limit takes the form of a lower bound on asset holdings $\underline{a}(t)$, such that asset holdings of household i must satisfy

$$a_i(t) \geq \underline{a}(t)$$

at every time t. The borrowing constraint satisfies $\underline{a}(t) < 0$, so that it is always possible for households to carry some debt.[9]

[8]These are flow probabilities, i.e. a job-finding rate $p(t)$ means that the probability that a worker who is unemployed at time t finds a job during the interval $[t, t+dt]$ is approximately $p(t)\cdot dt$, which becomes exact as $dt \to 0$.

[9]Technically, employed households face a looser borrowing constraint than unemployed households. However, assuming a single borrowing constraint rules out default following job loss. In any case, the borrowing constraint generally does not bind for employed households.

2.2 Household Problem under Repayment

There is no aggregate uncertainty, so the paths of all aggregate variables are known at time 0. Households face uncertainty about employment status $h_i(t)$ and asset holdings $a_i(t)$, and so we solve for household behavior given (h, a, t). Let $V(a,t)$ and $U(a,t)$ be the value functions of employed and unemployed households respectively. Employed households choose current consumption to maximize the current value Hamiltonian:

$$\rho V(a,t) = \max_c \{u(c) + V_a(a,t) \cdot \dot{a}_e + V_t(a,t) + s(t) (U(a,t) - V(a,t))\} \tag{2}$$

where V_a and V_t denote the partial derivatives of $V(a,t)$. Unemployed households similarly maximize the current value Hamiltonian:

$$\rho U(a,t) = \max_c \{u(c) + U_a(a,t) \cdot \dot{a}_u + U_t(a,t) + p(t) (V(a,t) - U(a,t))\} \tag{3}$$

I denote optimal consumption rules of employed and unemployed households as $c_e(a,t)$ and $c_u(a,t)$ respectively. These consumption rules satisfy:

$$u'(c_e(a,t)) \geq V_a(a,t) \tag{4}$$

$$u'(c_u(a,t)) \geq U_a(a,t) \tag{5}$$

These hold with equality when the constraint does not bind (always when $a > \underline{a}$), since unconstrained households are indifferent between consuming and saving and the margin. When the constraint binds, c is chosen s.t. $\dot{a} = \dot{\underline{a}}$.[10] Then the marginal utility of consumption is greater than the shadow value of wealth — intuitively, constrained households would like to borrow more, so they prefer consumption to saving at the margin.

2.3 Household Euler Equation

Restricting attention to unconstrained households, the consumption of employed households satisfies the Euler equation:

$$\underbrace{\frac{\dot{c}_e + s\,(c_u - c_e)}{c_e}}_{E[\dot{c}/c]} = \gamma^{-1}(r - \rho) + s \underbrace{\left[\gamma^{-1}\left(\frac{c_u^{-\gamma}}{c_e^{-\gamma}} - 1\right) - \left(1 - \frac{c_u}{c_e}\right)\right]}_{\text{precautionary motive}} \tag{6}$$

[10] We may have $\dot{\underline{a}} \neq 0$ as the borrowing constraint varies with changing macroeconomic conditions.

where $\dot{c}_e(a,t) = \frac{\partial}{\partial a} c_e(a,t) + \frac{\partial}{\partial t} c_e(a,t)$ is the rate of change in consumption by continuously employed households. The Euler equation for unemployed households is likewise:

$$\frac{\dot{c}_u + p\left(c_e - c_u\right)}{c_u} = \gamma^{-1}\left(r - \rho\right) + p\left[\gamma^{-1}\left(\frac{c_e^{-\gamma}}{c_u^{-\gamma}} - 1\right) - \left(1 - \frac{c_e}{c_u}\right)\right] \tag{7}$$

The derivations of (6) and (7) are given in Appendix B.1.1. The left-hand side of these expressions is expected consumption growth of a household. The term $\gamma^{-1}(r-\rho)$ captures intertemporal substitution in response to the interest rate: a high interest rate prompts households to defer current consumption, implying higher consumption growth.

The term in brackets captures the precautionary motive. We can write this term for employed households as $sT(c_u/c_e)$, and for unemployed households as $pT(c_e/c_u)$, where

$$T(x) = \gamma^{-1}\left(x^{-\gamma} - 1\right) - (1 - x) \tag{8}$$

Thus the precautionary motive is a function of the variability of consumption over an employment transition, times the probability of such a transition. $T(x) = 0$ at $x = 1$, i.e. there is no precautionary saving under perfect consumption smoothing, and $T(x)$ is strictly increasing as x moves away from 1.

To gain additional intuition for the precautionary motive term, we can take a second-order Taylor expansion around $x = 1$. The resulting expression can be written as:

$$sT\left(\frac{c_u}{c_e}\right) \approx \frac{1+\gamma}{2} \times Var\left(\frac{dc/dt}{c}\right) dt \tag{9}$$

That is, the precautionary motive is proportional to the instantaneous variance of consumption growth.[11] The term of proportionality is one-half the relative prudence of the utility function, as defined by Kimball (1990).

2.4 Aggregate Euler Equation

Aggregate consumption growth satisfies the aggregate Euler equation:

$$\frac{\dot{C}}{C} = \underbrace{(1-\chi)\,\gamma^{-1}\left(r - \rho\right)}_{\text{interest-rate substitution}} + \underbrace{T(\sigma_C^2)}_{\text{precautionary saving}} + \underbrace{\chi\left(\frac{p\Delta c(\underline{a}) - \ddot{\underline{a}}}{C}\right)}_{\text{constrained households}} \tag{10}$$

[11] $\frac{dc/dt}{c}$ is a random variable giving consumption growth over the time interval dt, taking the limit as $dt \to 0$. For details see appendix B.1.2.

where χ is the share of households that are constrained, $\Delta c(\underline{a}) = c_e(\underline{a}) - c_u(\underline{a})$ is the increase in consumption when a constrained household finds a job, and

$$T(\sigma_C^2) = \int \left[m_e \frac{c_e}{C} \cdot sT\left(\frac{c_u}{c_e}\right) + m_u \frac{c_u}{C} \cdot pT\left(\frac{c_e}{c_u}\right)\right] \tag{11}$$

is the consumption-weighted average of the precautionary saving terms of individual households.[12]

As discussed in the previous section, the precautionary terms are proportional to the instantaneous variance of consumption growth facing households. Thus the aggregate precautionary saving term $T(\sigma_C^2)$ is approximately equal to the consumption-weighted average variance of the growth rate of consumption facing unconstrained households. See appendix B.1.3 for details.

Observe that the interest-rate term in (10) is multiplied by the share of unconstrained households $1 - \chi$. This reflects that, whereas unconstrained households balance consumption volatility and intertemporal substitution, constrained households are restricted to simply consume their current income. The result is that future interest rates are discounted at the rate $1 - \chi$. This discounting makes the Euler equation less forward-looking, as emphasized by McKay, Nakamura, and Steinsson (2015). However, the precautionary saving term in (10) makes the aggregate Euler equation *more* forward looking. Thus whether incomplete markets make the Euler equation more or less forward-looking overall depends on the relative strength of these effects.

2.5 The Natural Rate of Interest

The aggregate Euler equation is critical to the dynamics of a New Keynesian model of the liquidity trap. To understand the source of this model's dynamics, it is useful to contrast the Euler equation above to the standard Euler equation under certainty:

$$\frac{\dot{C}}{C} = \gamma^{-1}(r - r^*) \tag{12}$$

r^* is often called the "natural rate of interest", which under complete markets is equal to the rate of time preference. Under the maintained assumption that the economy returns to its steady state in the long run, the path of consumption growth determines the path of consumption. In a simple model with no capital, consumption then immediately

[12] $m_e(a,t)$ and $m_u(a,t)$ give the mass of employed and unemployed households, respectively, with assets a at time t. The integral is taken over assets $a > \underline{a}$, excluding the point mass of constrained households with $a = \underline{a}$. For details of the derivation of the aggregate Euler equation, see Appendix B.1.3.

determines output and employment.

In this setting, a stylized way of capturing a demand shock is as a fall in the natural rate of interest r^*. If the central bank succeeds in setting $r = r^*$, the only effect will be a fall in the interest rate — in this case the central bank succeeds in fully offsetting the shock. However, if r does not fall enough, e.g. because of the zero lower bound on nominal interest rates, then $r > r^*$ will result. This implies a positive growth rate of output, which in turn implies a fall in current output.

Rearranging (10) slightly, we can express our aggregate Euler equation in terms of a "natural rate of interest" as well:

$$\gamma \frac{\dot{C}}{C} = r - \underbrace{\left[\rho - \gamma T(\sigma_C^2) - \gamma \chi \mu\right]}_{\text{Natural rate of interest}} \tag{13}$$

where $\mu = \left(p\Delta c(\underline{a}) - \underline{\ddot{a}}\right)/C - \gamma^{-1}\left(r - \rho\right)$ is the wedge due to binding constraints.

This expression for the natural rate differs in two respects from the complete markets benchmark. First, it contains a *precautionary* term arising from idiosyncratic consumption volatility due to incomplete markets. Further, as long as there is a non-trivial mass of households at the constraint, these households will mechanically consume their liquid wealth every period. Since these households are forced to reduce their debt holdings when the constraint tightens, I refer to this term as the *deleveraging* term.

Since both the precautionary and deleveraging terms are positive, their presence lowers the steady state rate of interest. One consequence of a lower interest rate is that the zero lower bound is a tighter constraint — it takes a smaller demand shock for it to bind. Moreover, any increase in income risk or tightening of borrowing constraints will lower the natural rate further, pushing the economy closer to the zero lower bound. In particular, if income risk increases and borrowing constraints tighten during recessions, this produces endogenous procyclical variation in the natural rate of interest. This is the basis of the amplification channel studied in detail in section 5.

2.6 Household Decision Rules

We can equivalently express household behavior in terms of saving decision rules $\dot{a}_e(a,t)$ and $\dot{a}_u(a,t)$. Figure 1(a) depicts these saving decision rules for employed and unemployed households in steady state.[13] Saving rates are decreasing in assets, because as households gain wealth the precautionary saving motive declines. Unemployed house-

[13] Steady state is defined in section 3.6, and baseline parameters given in section 3.5.

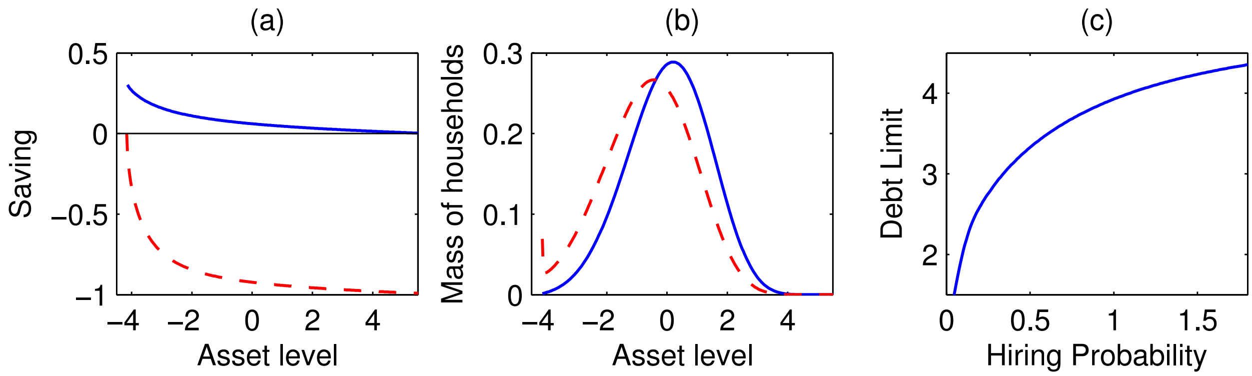

Figure 1: (a) Saving decision rules. (b) Asset distribution. (c) Borrowing Constraint. (employed (solid), unemployed (dashed))

holds dissave until they reach the borrowing constraint $\underline{a}$, whereas employed households accumulate assets up to a target level of assets $\bar{a}$.

The steady state asset distribution produced by these saving decision rules is shown in Figure 1(b). The figure depicts the mass of households conditional on employment status at each asset level. There is a point mass of unemployed households at the borrowing constraint, indicating that unemployed households borrow rapidly enough that they hit the borrowing constraint with non-vanishing probability.

Importantly, the saving decision rules are convex in assets. Intuitively, the sensitivity of consumption to changes in wealth is greater for poor households close to the borrowing constraint than for wealthy households. This convexity implies that an increase in the dispersion of asset holdings will decrease aggregate consumption. This makes aggregate spending dependent on the distribution of asset holdings. Since the curvature of the saving rule is greatest for unemployed households close to the borrowing limit, the share of households with low wealth (close to the constraint) is of particular importance.

The saving decision rules of households depicted in Figure 1(a) match several features of the data. First, they show that high debt households have a higher marginal propensity to consume out of wealth than low debt households, as found by Mian, Rao, and Sufi (2013). Also, the difference $\dot{a}_e - \dot{a}_u$ decreases as households become indebted, implying that consumption of high-debt households is more sensitive to job loss, as documented by Baker (2014).[14]

[14]However, these decision rules imply decreasing saving rates in wealth, whereas they are increasing in the data, as documented by Dynan, Skinner, and Zeldes (2004) and others. Since this paper focuses on precautionary saving and deleveraging at business cycle frequencies, it is not focused on lifecycle considerations and differences in lifetime wealth that are generally held to underlie this pattern.

2.7 Borrowing Constraint

I assume that indebted households make the decision to repay or default at every point in time t. If a household defaults, it suffers a fixed utility penalty D and is able to borrow again immediately. Thus the value function of an unemployed household that repays at time t is $U(a,t)$, whereas its value function under default is $U(0,t) - D$, and so an unemployed household will repay if and only if

$$U(a,t) \geq U(0,t) - D$$

Since the value function $U(a,t)$ is strictly increasing in a, there is a unique threshold level of assets below which unemployed households default. I assume that lenders set the borrowing constraint equal to this threshold level $\underline{a}(t)$, which is implicitly defined by

$$U(\underline{a}(t),t) = U(0,t) - D \tag{14}$$

Since employed households will never choose to hold assets $a < \underline{a}$, and unemployed households cannot choose to do so, no default will occur in equilibrium.[15]

Since the cost of default is fixed, endogenous variation in the borrowing constraint is driven by changes in the benefit from defaulting. This benefit is $U(\underline{a},t) - U(0,t)$, which we can write as $\int_{\underline{a}}^{0} U_a da$. Thus the borrowing constraint is a function of the marginal value of wealth to unemployed households. Any increase in the marginal value of wealth will tighten the borrowing constraint. Intuitively, if there is a greater benefit from defaulting, lenders must tighten the borrowing constraint to prevent defaults.

One of the chief determinants of the marginal value of wealth for unemployed households is the future path of the job-finding rate p. When future p is high, households expect that they will not be unemployed for very long, implying greater lifetime wealth and a lower marginal value of wealth. This is the mechanism by which high unemployment causes tighter borrowing constraints.

Figure 1(c) depicts the borrowing limit $|\underline{a}(p)|$ for various levels of the job-finding rate p.[16] The figure illustrates that borrowing constraints can vary substantially with the hiring probability. From the beginning of 2008 to the end of 2009, the quarterly job-finding rate in the U.S. fell from about 1.9 to 0.75, which according to Figure 1(c) would tighten

[15] There is also a much lower default threshold for employed households defined by $V(\underline{a}^e,t) = V(0,t) - D$. However, since employed households always choose to accumulate assets, no household will ever hold $a < \underline{a}$, and this constraint will never be reached.

[16] Figure 1(c) uses parameters given in section 3.5, and solves the household problem for various levels of p, holding other variables constant. Thus this is a partial equilibrium comparison.

the borrowing constraint by over 20%.

3 Equilibrium

Now that we have discussed the household problem, I describe the supply side of the model and define equilibrium.

3.1 Firms

Final goods are produced from intermediate goods by a final goods firm using a Dixit-Stiglitz aggregation technology.[17] This yields demand for intermediate good i of

$$y_i = Y\left(p_i/P\right)^{-\epsilon} \tag{15}$$

where Y is aggregate demand.

There is a measure 1 of identical intermediate good firms that face demand function (15). Firms produce using production function

$$y_i(t) = z(t) \cdot \left(n_i(t)\right)^{1-\alpha}$$

I assume that firms pay a negligible (in the limit, zero) hiring cost on new workers, and so prefer to retain existing workers rather than hire new workers when possible. Firms lose workers at a constant exogenous separation rate s.[18]

I assume that the resulting path of employment $n(t)$ is continuously differentiable, so that $\dot{n}(t)$ is well-defined at every $t \geq 0$.[19] Then the job-finding rate facing households is:

$$p(t) = \frac{\dot{n}(t) + s(t)n(t)}{1 - n(t)}$$

I consider firm behavior under two pricing regimes: fixed prices and fully flexible prices. The fixed price regime will be the baseline case and may be interpreted as a demand-constrained environment with anchored inflation expectations. The flexible price case defines a benchmark that serves as the central bank policy target.

[17] For details, see appendix B.2.

[18] A fixed separation rate is consistent with the data: Shimer (2012) finds that variation in the separation rate is "quantitatively irrelevant" over the years 1990 – 2010.

[19] Later I will allow for an unanticipated shock at $t = 0$, and therefore only require the right derivative $\dot{n}(t)$ to exist at $t = 0$.

Flexible Prices. Suppose that firms may adjust their prices and choose any (p_i, y_i) consistent with (15) at every t. Further suppose that the government provides a production subsidy to firms, financed by a lump-sum tax, that exactly offsets underproduction due to market power. Then optimal production by firms satisfies:

$$(1-\alpha)\, z(t)\, (n_i(t))^{-\alpha} = w(t) \tag{16}$$

To focus attention on the demand side, I assume that productivity z and the wage w are constant over time. Thus optimal hiring corresponds to a constant level of employment given by:

$$n^* = \left(\frac{(1-\alpha)\, z}{w}\right)^{\alpha} \tag{17}$$

We may assume that the prevailing wage in the economy is determined by some unmodelled bargaining process, in which there is some disutility from labor that determines households' willingness to work at a given wage. If this were modelled directly, the equilibrium wage would depend on the asset distribution, since wealthy households would withdraw from the labor force when their marginal utility of wealth fell sufficiently that the equilibrium wage no longer compensated them for their labor. I take the equilibrium wage as given to avoid these effects, since the primary focus of this model is on the demand side. The result is that there is a constant flexible price "natural" level of employment, which we can take to be the target of policymakers.[20]

Fixed Prices. Now suppose that firms' prices are fixed at $p_i(t) = 1$, which implies that the aggregate price level is $P(t) = 1$. As is standard in New Keynesian models, firms are required to meet the demand that they face at this price. Thus firm i chooses a path of $n_i(t)$ that satisfies

$$d_i(t) = z(t) \cdot (n_i(t))^{1-\alpha} \tag{18}$$

where $d_i(t)$ is demand for the firm's output. Thus at every point in time, firms set output equal to demand. I further assume that firms will be able to hire sufficient workers to meet the demand they face, which will be true in all the cases we consider.

I assume that wages are fixed at the equilibrium level $w(t) = \bar{w}$. This implies that wages are unaffected by fluctuations in demand. Thus flow profits are

$$\pi_i(t) = z_t\, (n_i(t))^{1-\alpha} - \bar{w} n_i(t) \tag{19}$$

[20] A further complication would arise if firms bargained directly with each household, since each households' reservation wage is a function of its wealth as well as the aggregate job-finding probability.

I assume that households own firms, so that households' nonlabor income is $e = \bar{e} + \pi$.

Notes on assumptions. The assumption of fixed prices is a useful simplification that allows us to abstract from price adjustment and expectation formation, and focus on the key mechanism of demand rationing when the real interest rate cannot fall. The model could easily be extended to include partially flexible prices. In this case, employment losses at the zero lower bound would be greater, because the low output would cause deflation, raising the real interest rate.[21] Thus fixed prices is a conservative assumption. Moreover, this assumption is consistent with the lack of deflation in the U.S. in the years following 2008, which suggests that inflation expectations were well-anchored during this period.[22]

Although many researchers consider sticky wages a promising explanation for employment volatility,[23] with fixed prices output is wholly determined by demand anyway. The main consequence of fixed wages here is to avoid countercyclical profits. Besides being inconsistent with the data, countercyclical profits would reduce unemployment risk by increasing the relative income of unemployed households during downturns.[24]

3.2 Equilibrium under Fixed Prices

Under fixed prices, there is no inflation and the real interest rate $r(t)$ is set by the central bank. Exogenous variables $\{\bar{e}(t), z(t), s(t)\}$ are likewise given. Now we need an asset market clearing condition to close the model. Let $m_e(a,t)$ and $m_u(a,t)$ be the mass of employed and unemployed households with assets a at time t. m_e and m_u are related to employment n by

$$n(t) = \int_a m_e(a,t)da \tag{20}$$

$$u(t) = 1 - n(t) = \int_a m_u(a,t)da \tag{21}$$

[21] Eggertsson and Krugman (2012) calls this result the *paradox of flexibility*. Note that despite fixed wages, marginal costs are procyclical because the production function is concave.

[22] This approach to modeling demand rationing is similar to that taken by Korinek and Simsek (2014), Kocherlakota (2012), and Hall (2011).

[23] See, e.g., Hall (2005), Pissarides (2009), and Galí (2011).

[24] If inverse labor supply was $w(n)$, then fluctuations in employment would affect profits according to $d\pi/dn = (f_n - w) - w_n n$. Since $w \approx f_n$ in the neighborhood of n^*, $w_n > 0$ implies countercyclical profits, while fixed wages imply negligible procyclical fluctuations in profits.

In equilibrium, aggregate assets are always zero:

$$\int_a a \cdot m_e(a,t)da + \int_a a \cdot m_u(a,t)da = 0 \tag{22}$$

The asset distribution evolves according to the law of motion

$$\dot{m}_e = pm_u - sm_e - \frac{d}{da}(m_e \dot{a}_e) \tag{23}$$

$$\dot{m}_u = sm_e - pm_u - \frac{d}{da}(m_u \dot{a}_u) \tag{24}$$

To interpret (23) and (24), note that the change in the mass of households at a particular point equals flow in minus flow out of that point. The flow through a point by households that do not change employment status is $\frac{d}{da}(m\dot{a})$. Note also that we can obtain the law of motion of aggregate labor $\dot{n} = p(1-n) - sn$ by integrating (23) over a.

Definition 1 (Equilibrium)**.** Given a path of $\{z, \bar{e}, r, s, D, w\}$ and initial asset distribution $m_e(a,0)$ and $m_u(a,0)$, an equilibrium is a path of $\{m_e, m_u, V, U, c_e, c_u, \dot{a}_e, \dot{a}_u, e, \pi, p, \underline{a}, n, u\}$ that satisfies (1) - (14), (19), and (20) - (24).

3.3 Multiple Equilibria for Fixed Path of Interest Rates

Consider the case with a fixed uncontingent path of the real interest rate $r(t)$, which remains at the steady state value after some point. Then there may be multiple equilibrium paths that converge asymptotically to the steady state. This is in contrast to the standard New Keynesian model, in which a given path of the real interest rate uniquely determines the path of output. To see why this occurs, consider a path of employment that converges slowly to n^* from below. Then the job-finding rate will be lower than its steady state value along this path. Since households face greater unemployment risk, they save more and spend less which lowers demand. Thus a lower path of employment produces a lower path of demand.

For this mechanism to result in multiple equilibria, a low job-finding rate must reduce demand by a sufficiently large amount. To state this condition for precisely, consider an economy in the neighborhood of the steady state, with fixed $r(t) = r^*$. An equilibrium is a path of the variables (n, p, μ), where μ captures the asset distribution. Suppose that variations in μ have second-order effects on demand compared to changes in (n, p), so that we can treat the asset distribution as fixed. Then equilibrium (n, p) satisfies:

$$\dot{n} = p(1-n) - sn \tag{25}$$

$$y(n) = d(n, \vec{p}) \tag{26}$$

where $\vec{p}(t)$ is the full path of $p(\tau)$ for $\tau \geq t$. Here (25) is the law of motion of employment, or equivalently the definition of the job-finding rate, and (26) is the condition that output equals demand.

To set a lower bound on the future path of p necessary to sustain a given level of demand, we replace $\vec{p}$ with a constant future path of p. Then $y(n) = d(n, p)$ defines a schedule in (p, n) space, corresponding to the level of employment consistent with goods market clearing for a particular sustained job-finding rate. I call this the *steady state demand* curve. Further, (25) can be used to define a curve of (p, n) pairs at which $\dot{n} = 0$. These curves intersect at the steady state.

The steady state demand curve defined by (26) is upward sloping in (p, n) space. Its slope is:

$$\frac{dn}{dp} = \frac{d_p}{y_n - d_n}$$

The numerator is positive because higher p lowers income risk facing households, and reduces precautionary saving. The denominator is positive because $y_n \approx \bar{w}$ in the neighborhood of the steady state, while $d_n < \bar{w}$. The latter holds because if an unemployed household finds a job, its income rises by $\bar{w}$, but some of this additional income is used to increase its net saving. Therefore demand rises by less than the wage.

Now we have a simple condition for the existence of multiple equilibria: the steady state demand curve must must intersect the curve $\dot{n} = 0$ from below in (p, n) space, i.e. must have a greater slope. We can write this condition as:

$$\frac{d_p}{y_n - d_n} > \frac{p+s}{1-n} \tag{27}$$

Figure 2 depicts this condition in the two-dimensional phase space of the system.[25] Here $\hat{n}$ and $\hat{p}$ refer to the difference between the variable and its steady state value. Panel (a) shows the case of a unique equilibrium. In this case, the steady state demand curve lies above the $\dot{n}$ curve, where $\dot{n} < 0$. In this case, demand is not very sensitive to the job-finding rate p, and thus a large decrease in p is necessary to a lower level of n. This

[25]The phase space of the whole model has much higher dimension, since it also includes the asset distribution. This phase diagram is constructed assuming second-order effects on demand from the variation in the asset distribution.

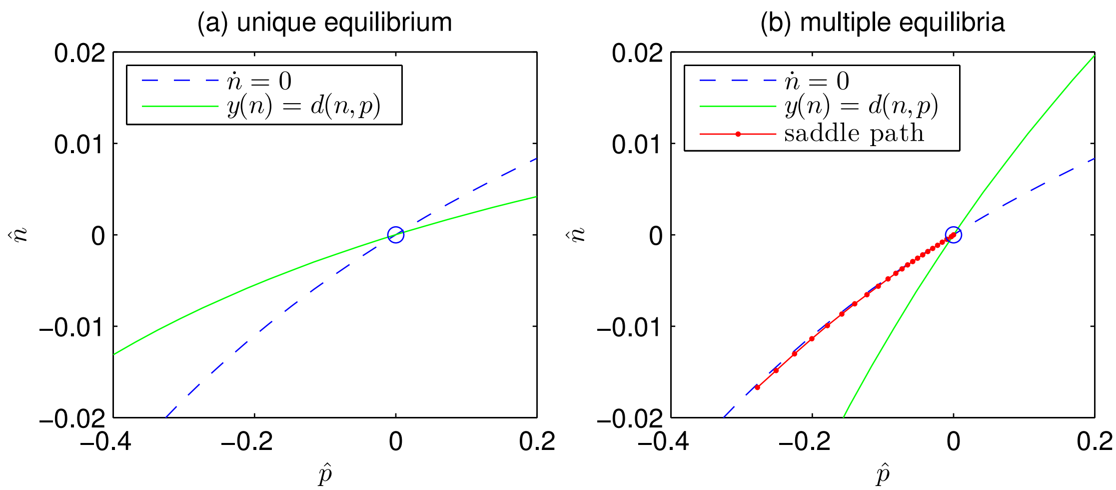

Figure 2: Convergence to steady state.

implies that any $n < n^*$ must be accompanied by a low p, which results in $\dot{n} < 0$, and is therefore inconsistent with convergence to the steady state.

By contrast, panel (b) shows the steady state demand curve under the baseline calibration described in section 3.5. Now condition (27) is satisfied, and there is a convergent saddle path that lies between the two curves. Intuitively, if demand is highly sensitive to p, a value of $n < n^*$ requires only a small fall in the future path of p, and thus is consistent with $\dot{n} > 0$. The equilibrium path is depicted in panel (b), and lies between the two curves.

Comparison with other models. Why are there multiple equilibria in this model, but not in other models with heterogenous agents and precautionary saving? The difference is how I model cyclical variation in income. In the models of Guerrieri and Lorenzoni (2011) and McKay, Nakamura, and Steinsson (2015), the transition probabilities between employment and unemployment (more precisely, between hgh and low idiosyncratic productivity) are constant over the business cycle, while the wage is procyclical. Thus a recession in these models means a lower wage, but no greater probability of employment. This actually *decreases* income risk facing households, because the difference in income between the high and low productivity states is diminished. Thus in their models the curve defined by (26) is flat or even somewhat downward-sloping, resulting in a unique path of equlibrium for fixed path of interest rates. By contrast, my model assumes a fixed wage and a changing transition probability, and thus generates procyclical income risk.

The multiple equilibria in this model are similar to that in Chamley (2014), in which uncertainty about the ability to sell goods in the future may be self-fulfilling. This is analogous to uncertainty about future job prospects affecting saving behavior of households. The mechanism is also similar to the high-unemployment low-asset value equilibrium in Heathcote, Storesletten, and Violante (2009). By contrast, other recent work that focuses expliclity on precautionary responses to procyclical unemployment risk, such as Challe et al. (2014), Ravn and Sterk (2013), and Caggese and Perez (2013), do not discuss multiple equilibria, because they focus on cases where the interest rate is flexible and so the central bank can select a particular equilibrium, as we shall see below.[26]

Since condition (27) is satisfied in our baseline calibration in the neighborhood of the steady state, there will in general be multiple equilibrium paths that converge to the steady state for given path of exogenous variables including real interest rates $r(t)$. To specify a unique equilibrium path, it is sufficient to specify $n(0)$, which will allow us to calculate a unique path of equilibrium for given $r(t)$. Alternatively, if the central bank sets a condition interest rate rule, it may eliminate some equilibria, potentially specifying a unique equilibrium path. We turn to central bank policy next.

3.4 Central Bank Policy Rule

Definition 1 defines equilibrium for given exogenous variables and boundary condition. However, it leaves open the question of what path of interest rates the central bank sets. In this section we discuss the central bank policy rule.

Perfectly Flexible Interest Rates. First suppose that there are no restrictions on the path of interest rates that the central bank can select. Suppose that the central bank tries to replicate the flexible price employment rate n^* defined by (17).[27] With flexible interest rates, the central bank will always be able to hit its policy target, and the employment rate satisfies $n = n^*$. This also defines a constant job-finding rate of $p^* = sn^*/(1 - n^*)$. Given this path of employment and job-finding rates, there will be a unique path of the real interest rate that is consistent with equilibrium.

The reason that there are not multiple equilibria in this case is because the central bank does not set a single path of the real interest rate, but rather sets a contingent path of r to

[26] I do not know whether these models would generate multiple equilibria for a given real interest rate, since it depends on whether the analogous condition to (27) holds in their models.

[27] This policy rule is equivalent to setting the interest rate equal to the Wicksellian natural rate, or inflation targeting if prices were partially flexible. This would be optimal if the only friction were sticky prices. However, in this model the flexible-price equilibrium is not Pareto optimal because of the presence of incomplete markets. We may interpret this policy rule as a central bank that limits itself to short-run stabilization, and does not seek to correct inefficiencies arising from long-term structural features of the economy.

achieve its target level of n^*. Since households know that the central bank will be able to achieve $n = n^*$, they expect this and it is the unique equilibrium.

Lower Bound on Interest Rates. Now suppose that there is a lower bound on the interest rate $\underline{r}$, so that only policy paths that satisfy $r(t) \geq \underline{r}$ are possible. Then it might not be possible for the central bank to hit its target job-finding rate $n = n^*$. In particular, if the flexible interest rate equilibrium requires the interest rate to fall below $\underline{r}$ at any point, then this equilibrium violates the constraint.

We must now modify the policy rule of the central bank. Instead of assuming a fixed employment rate $n(t) = n^*$ and a variable interest rate, suppose that at every point in time *either* the central bank has successfully hit its target $n(t) = n^*$ and the interest rate takes on some value $r \geq \underline{r}$, *or* the interest-rate constraint is binding and the central bank cannot hit its target, meaning that $r = \underline{r}$ and $n(t) \leq n^*$. Note that the central bank can always kill off an excessive boom by raising interest rates, so we don't need to worry about $n > n^*$. This yields the *constrained policy rule*:

$$r(t) \geq \underline{r} \text{ and } n(t) = n^* \quad \text{OR} \quad r(t) = \underline{r} \text{ and } n(t) \leq n^* \tag{28}$$

The assumption that the central bank will not allow $n(t) > n^*$ implies a lack of commitment on the part of the central bank. It may be the case that the lower bound on the interest rate causes a period of low employment which the central bank could reduce if it could credibly promise to allow $n(t) > n^*$ after the liquidity trap has concluded. In section 6 I will analyze what happens if we allow such forward guidance.[28]

Equilibrium selection rule. Given the lower bound on interest rates, the central bank may not be able to achieve $n = n^*$ at all times. To fix intuitions, consider the case that the economy experiences an unanticipated adverse demand shock at $t = 0$ that causes $r < \underline{r}$ in the flexible rate equilibrium, i.e. the lower bound on the interest rate binds. Suppose further that the economy permanently exits the liquidity trap at some future point T^*.[29] Then from the policy rule (28), we know that $\forall t \leq T^*$, $r(t) = \underline{r}$ and $n(t) \leq n^*$, and $\forall t \geq T^*$, $r(t) \geq \underline{r}$ and $n(t) = n^*$. The period $t < T^*$ corresponds to the liquidity trap, when employment is below its target level and the interest rate is at the lower bound. The

[28]This no commitment assumption is similar to the baseline case considered by Werning (2011), who considers a liquidity trap that ends at a fixed time T, and assumes that the central bank implements the no commitment equilibrium for $t \geq T$.

[29]This formulation is similar to that used in Werning (2011) and related papers in the liquidity trap literature, except that in my model the date of exit from the liquidity trap T^* is endogenous.

period $t \geq T^*$ is after the liquidity trap, when the central bank can achieve its employment target.

However, in general there will be multiple T^* consistent with equilibrium. A later T^* would correspond to a larger initial fall in $n(0)$ and a slower recovery. This is analogous to slower convergence to the steady state in the fixed interest rate case analyzed in section 3.3. Intuitively, if people expect a slower recovery, they will reduce spending all along the path of recovery, so that the slow recovery is self-fulfilling.

In my baseline experiments, I will assume that the economy follows the equilibrium with the highest initial level of employment $n(0)$ consistent with the central bank's no-commitment policy rule (28). In section 6 I consider the possibility of worse equilibria, corresponding to pessimistic expectations about the pace of recovery, and better equilibria, corresponding to forward guidance. None of these will require a different path of the real interest rate, but will work entirely through households' expectations of the joint path of (p, n).

3.5 Calibration

I use the following parameters in the baseline calibration:

α	$\bar{w}$	$\bar{e}$	z	s	γ	ρ	D
1/3	1	0.05	1.46	0.1	1	0.40	3.30

Table 1: Baseline Calibration

I normalize the wage to 1, and set α so that labor income is 2/3 of total output. I set the quarterly separation rate to 0.1, and set productivity to target a steady state quarterly job-finding rate for unemployed households of 1.35. These correspond to the average monthly separation and job-finding rates for the US economy during 1951 – 2003, as reported by Shimer (2005).[30] Together with the job separation rate, this implies a steady state unemployment rate of 6.9%.

The coefficient of relative risk aversion $\gamma = 1$ is a standard choice in business cycle models, e.g. in Blanchard and Galí (2010). It is also the middle of the range of estimates reported by Gourinchas and Parker (2002) based on their analysis of household behavior in the Consumer Expenditure Survey and Panel Study of Income Dynamics. Note that this is a more conservative estimate of risk aversion than assumed by most papers that have focused on precautionary savings, such as Guerrieri and Lorenzoni (2011).

[30] A job-finding rate of 1.35 implies a 74.1% total chance of finding a job in a quarter.

I set households' time-discount rate to a target steady state interest rate of 0.25% on an annual basis. This target was chosen so that the zero lower bound is effectively binding in the steady state. While this calibration is mainly chosen for tractability, such an occurrence is not completely unreasonable, given that Japan has experienced zero short-term policy rates for most of the last 25 years, as have the U.S. and Europe since 2008. Some commentators have suggested that a combination of well-anchored inflation expectations, low population growth rates, and slow technological progress have lowered the natural (flexible price) rate of interest to zero or below, a situation known as "secular stagnation."[31] If they are right, this calibration could be literally correct. Nevertheless, the qualitative results of the model do not depend on this assumption.

The default penalty is set so that the steady state borrowing constraint equals 4.16 times labor income, as in Guerrieri and Lorenzoni (2011). This produces an analogous asset distribution across households, but shifted to the left. The shift is because I assume that assets are in zero net supply, so that the distribution is centered at zero, whereas Guerrieri and Lorenzoni (2011) assume positive net aggregate asset holdings of households due to a supply of government bonds.[32]

3.6 Steady State

A steady state equilibrium is a fixed price equilibrium that satisfies $\dot{m}_e = 0$ and $\dot{m}_u = 0$, i.e. that has a stationary asset distribution. For given exogenous variables $\{z, e, s\}$, the equilibrium conditions define a continuum of steady state equilibria, corresponding to various pairs $\{r, n\}$. We may think of this schedule as a menu of employment targets from which the central bank may pick, with higher n corresponding to higher r.[33] The central bank then chooses an employment rate target, which determines the corresponding steady state interest rate. The steady state equilibrium with target employment rate n^* corresponds to the flexible price steady state equilibrium, and the corresponding interest rate is the natural rate of interest in steady state. The flexible price steady state asset distribution and saving decision rules were shown in Figure 1(a) and (b).

[31]The concept of secular stagnation was first proposed by Hansen (1939). Eggertsson and Mehrotra (2014) present a model of secular stagnation.

[32]However, the direct comparison is a bit tricky, because Guerrieri and Lorenzoni (2011) have households that differ in labor productivity.

[33]Higher n implies a higher job-finding rate p, and therefore lower unemployment risk. This reduces precautionary saving, decreasing steady state asset demand and raising the equilibrium interest rate.

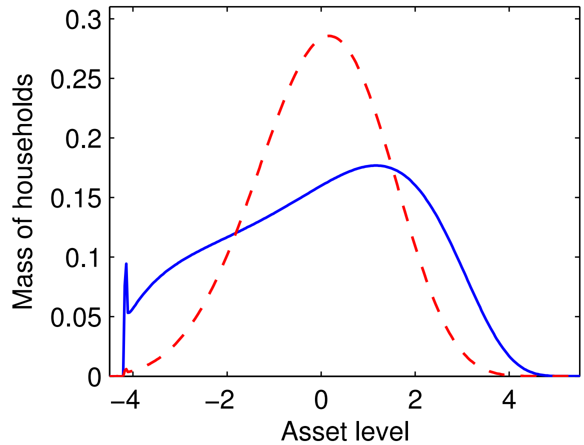

Figure 3: High-debt distribution (solid) vs. steady state (dashed).

4 Deleveraging Episode

Total household debt increased rapidly in the years before 2007, and fell during the recession that followed. Motivated by these facts, several researchers have proposed that the severity of the 2007 – 2009 recession and the weakness of the recovery were due to these high levels of household debt. For instance, Mian and Sufi (2010) find that counties with higher household leverage in 2006 experienced larger declines in employment, and Dynan (2012) argues that high household leverage contributed to weak consumer spending throughout the recovery.

I model a deleveraging episode by supposing that the economy enters $t = 0$ with a greater number of highly indebted households than in steady state. Since there are zero net assets, this corresponds to an increase in the dispersion of asset holdings. This produces a period of adjustment as the economy returns to steady state. Since greater asset dispersion reduces aggregate household spending, this implies a period of weak demand.

We can interpret this experiment as the unwinding of excessive debts accumulated in an unmodeled earlier period. For instance, perhaps there was a credit boom in $t < 0$, enabled by a housing bubble. Then at time t there was an unanticipated correction to credit conditions, e.g. due to the bursting of the housing bubble, requiring a transition to a new steady state. I explicitly model such a shock in Appendix A.1, but for now I simply focus on the dynamics of an economy that finds itself with excessive debt, whatever the reason.[34]

[34]One can also think of this as a redistribution shock that increases inequality — i.e. an unanticipated transfer of funds at time 0 from some indebted households to wealthy houses.

4.1 Fixed Price Equilibrium

Suppose the economy enters $t = 0$ with initial asset distribution $\{m_e, m_u\}$ that is a mean-preserving spread of the steady state distribution. We are interested in the dynamics of the fixed price equilibrium in a liquidity trap. The key feature of a liquidity trap is that the interest rate cannot fall, and so we fix the interest rate at its steady state level. The greater dispersion of asset holdings reduces demand, as discussed in section 2.6. Since the interest rate cannot fall to accomodate this lower demand, employment falls below its steady state level.

The mean-preserving spread of the initial asset distribution is chosen to produce initial employment 5 pct. points below steady state. This is equal to the rise in the unemployment rate in the U.S. between January 2008 and October 2009, i.e. from the official beginning of the recession until the trough of the labor market. Figure 3 depicts this asset distribution, with the steady state distribution shown for comparison.[35]

Figure 4 depicts the resulting equilibrium path under the baseline equilibrium selection rule discussed in section 3.2. The initial fall of employment of 5 pct. points is followed by a slow recovery of employment. Employment recovers to 1.4 pct. points below steady state after ten quarters, and to 0.7 pct. point after twenty quarters. Employment does not fully recover to its steady state level until 60 quarters, implying depressed output for 15 years from the beginning of the recovery. The cumulative employment loss is equivalent to 43.7 pct. points of employment for one quarter, or 10.9 pct. points of employment for one year.

The job-finding rate moves in tandem with the employment rate. This is because employment remains low for some time, implying a low rate of job growth $\dot{n}$. The job-finding rate is mechanically related to employment growth by the law of motion of employment $\dot{n} = p(1 - n) - sn$. As demand recovers, the job-finding rate rises in tandem with employment as firms increase hiring to boost production. Thus any persistent period of low employment implies a low job-finding rate.

The low job-finding rate amplifies the initial fall in demand. This happens for two reasons. First, the low job-finding rate raises income volatility facing households, increasing precautionary savings. Second, the low job-finding rate increases the incentives for default, which endogenously tightens the borrowing constraint facing households. The latter effect can be seen in the lower right panel of Figure 4, which shows that the borrowing limit tightens by about 4.5% relative to the steady state. Since both of these factors lower

[35] To minimize issues arising from the behavior of households with debt greater than the borrowing constraint, I use a mean-preserving spread that does not affect the support of the distribution. Appendix B.4 describes how this mean-preserving spread is calculated.

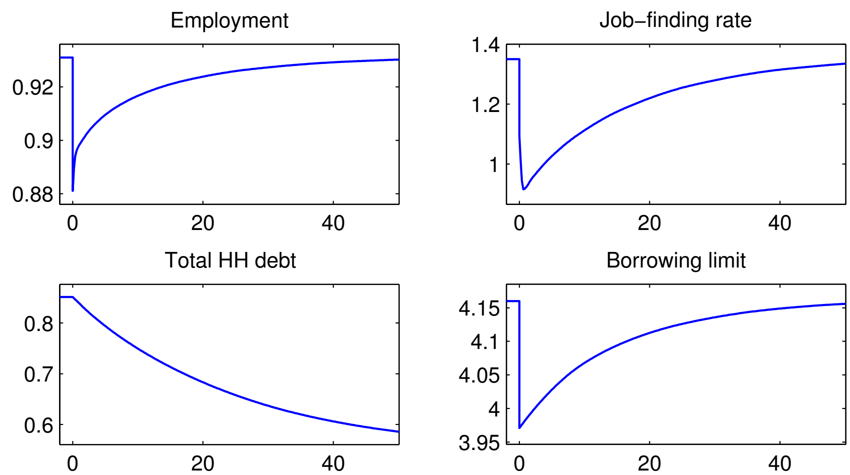

Figure 4: Equlibrium with initial high debt.

demand, they make the initial fall in employment larger than it would otherwise be, i.e. they act to amplify the initial shock.

4.2 Euler Equation Decomposition

During the recovery, demand falls both because constrained households are forced to reduce spending, a *deleveraging effect*, and because the low job-finding rate raises precautionary saving by unconstrained households, a *precautionary effect*. In this section we compute the relative contributions of these two effects to the total fall in demand, and therefore employment.

The path of demand is determined by the aggregate Euler equation

$$\frac{\dot{C}}{C} = \gamma^{-1}(r - r^*)$$

Thus slow employment growth during the recovery is equivalent to a low natural rate of interest r^*. We can use equation (10) to decompose the fall in the natural rate into deleveraging and precautionary components, which also implies a decomposition of the initial fall in employment.

Figure 5 shows the path of both components of the natural rate in the course deleveraging. The natural rate falls sharply at first, returns to around -1% after about one quarter, and then slowly returns to its steady state level. Most of the sharp initial fall is due to a reduction of spending by constrained households. Before the shock, there is an initial mass of households near the constraint (Figure 3). When the constraint endogenously tightens, these households are forced to deleverage, causing an immediate drop in de-

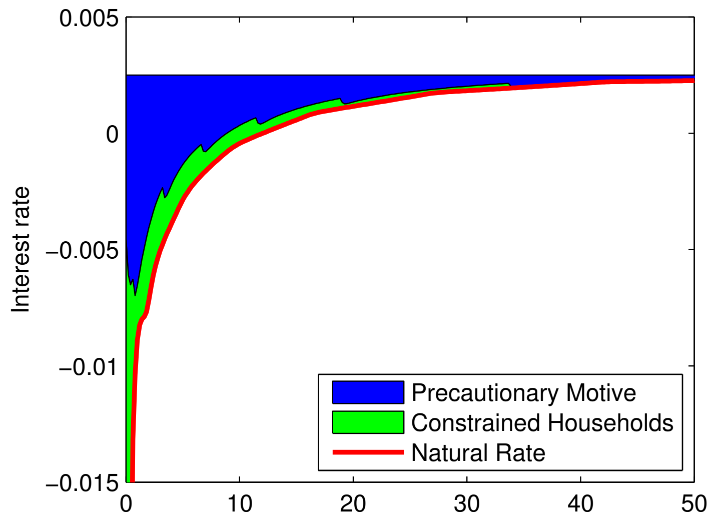

Figure 5: Decomposition of natural rate.

mand.[36] After this initial deleveraging, which lasts about 1 quarter, demand remains depressed for a time. The precautionary effect accounts for the majority of the reduction in the natural rate for the remainder of the crisis.

The total log consumption gap at any point in time is the integral of the gap between the interest rate and the natural rate forward in time

$$\log(C(t)) - \log(C^{ss}) = -\gamma^{-1} \int_t^\infty (r(t) - r^*(t))dt$$

This integral corresponds to the shaded area between the interest rate and natural rate lines in Figure 5. Since we can decompose the natural rate into deleveraging and precautionary components, this expression implies a decomposition of the output gap between these components at each point in time. This calculation reveals that 55% of the initial fall in aggregate demand is due to precautionary saving effects, whereas 45% is due to deleveraging. Thus both deleveraging and precautionary saving contribute to the initial fall in employment, but the contribution of precautionary saving is somewhat larger.

This calculation is somewhat misleading, since most of the employment loss due to deleveraging lasts only one quarter. A better measure is cumulative output loss, which more closely corresponds to the welfare loss during a recession. To calculate this, we decompose the source of the output gap at every point in time, and then sum over time. This calculation reveals that 81% of the cumulative output loss is due to the precaution-

[36] To avoid complications arising from endogenous default, I assume that these households consume a small fixed amount, and then must pay down their debts as quickly as possible.

ary motive, while 19% is due to binding borrowing constraints. This decomposition suggests that analyses of deleveraging episodes that focus exclusively on the behavior of constrained households neglect a significant mechanism through which excessive debt affects household spending.

4.3 Measure of Amplification

We now turn to determining the magnitude of the amplification from endogenous unemployment risk. Amplification arises from the interaction of endogenous variation in the job-finding rate with the precautionary behavior of households. Thus a natural comparison case is the complete-markets benchmark — i.e. the baseline model with the additional assumption that households are able to fully insure against employment shocks. Since households are risk-averse, the result will be full risk-pooling, so that all households enjoy the same level of consumption and zero assets. In this case, the aggregate Euler equation is the standard complete markets Euler equation:

$$\frac{\dot{C}}{C} = \gamma^{-1}(r - r^*)$$

and the dynamics of demand, and therefore employment, are determined by the path of $r - r^*$. The model is otherwise identical.

The main obstacle we face in computing a complete markets comparison case is choosing the appropriate shock. The baseline financial shock will not affect the complete markets benchmark at all — since no household wants to borrow, a tighter borrowing constraint makes no difference. The standard way to model a demand shock in this setting is as an exogenous change in the natural rate of interest r^*.[37] Thus we would like to calculate a path of r^* that corresponds to a reduced-form representation of the baseline credit shock in the full incomplete markets model.

To do so, we first compute the flexible price equilibrium following the credit shock. This implies a path of the interest rate that stabilizes output and the job-finding rate. This path corresponds to another notion of the "natural rate" — the interest rate that the central bank would have to set to fully offset the baseline demand shock. We can take this path of the interest rate as r^* in the complete markets benchmark, i.e. as a reduced form representation of the initial demand shock in the absence of feedback from endogenous variation in unemployment risk to demand.

Given this path of the natural rate, we can now compute the complete markets com-

[37] For instance, this is how demand shocks are modeled in Werning (2011).

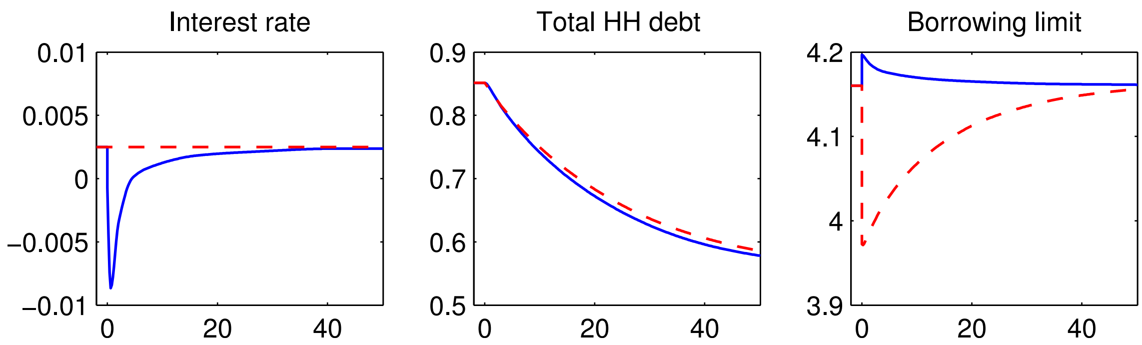

Figure 6: Deleveraging with flexible prices (solid) vs. fixed prices (dashed).

parison case. The central bank is again constrained by the lower bound on r, but the natural rate r^* now follows the path defined above.[38] The aggregate Euler equation then implies a path of demand and output, and this path of output implies a path of employment. Comparing this to the path of employment from the full model gives a measure of amplification from endogenous employment risk. Further, we can compare the path of r^* to the "natural rate" computed in the previous section, which allows us to decompose the amplification into deleveraging and precautionary components.

Flexible Price Benchmark. What would happen if the central bank were able to adjust the interest rate during the deleveraging episode? In this case the central bank would set r to replicate the flexible price equilibrium. Since the distribution of assets does not affect the incentives for production, the firm optimality condition (16) implies a constant level of employment, and therefore a constant job-finding rate. The interest rate would adjust to produce the demand necessary to sustain this level of production.

This equilibrium is shown in Figure 6. The left panel depicts the path of the interest rate. In the flexible price model, the interest rate falls initially by about 110 bp to accommodate an initial burst of deleveraging by highly indebted households. The interest rate then quickly recovers to about 25 bp below its steady state value after 5 quarters. It remains slightly depressed for some time, as the economy slowly transitions to its new stationary asset distribution. This indicates that the total reduction in demand due to the necessary deleveraging process is not so great, since it requires only a small fall in the interest rate to accommodate.

As the center panel of Figure 6 makes clear, the reduction in total household debt occurs somewhat faster under flexible prices than in the fixed price case. In particular, after 40 quarters excess household debt has fallen by 84% in the flexible price case compared to

[38]Here r^* is equivalent to the households' time discount factor ρ

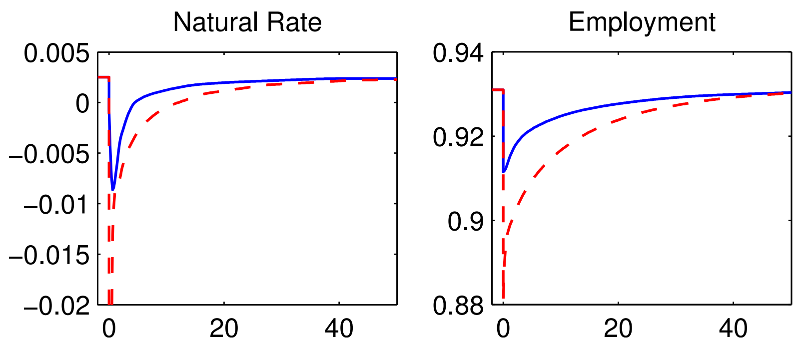

Figure 7: Complete markets (solid) vs. incomplete markets (dashed) models.

81% in the fixed price case. This happens because labor market conditions are stabilized under flexible prices, and so households are in a better position to pay down their debts. Under fixed prices employment falls and remains low throughout the transition, so that a higher fraction of highly indebted households are unemployed. These households do not reduce their debts while they are unemployed, and so high unemployment slows down the pace of deleveraging.

Finally, as the right panel indicates, borrowing constraints do not tighten in the flexible price case. The tightening of the borrowing constraint in the fixed price case is entirely due to poor labor market conditions. Since output and hiring are stabilized in the flexible price case, there is no endogenous feedback to credit conditions, and so borrowing constraints fail to tighten. In fact, they loosen slightly, because lower interest rates reduce the cost of repayment.

Comparison to Complete Markets Model. As described above, we can use the path of the interest rate in the flexible price case to obtain a measure of amplification. Treating the flexible price interest rate as a reduced form shock to the natural rate of interest in the complete markets model, we compute the reduction in demand attributable to increased asset dispersion, with no feedback from labor market conditions. We then compute the resulting path of demand arising from this reduced form shock, and compare it to the path of demand from the full model with endogenous income risk. The difference is a measure of the magnitude of amplification from this channel.

Figure 7 presents this comparison. The left panel compares the path of the natural rate of interest in each case. Overall, the natural rate falls farther and remains lower in the incomplete markets model. This reflects amplification arising from endogenous employment risk: since the central bank does not stabilize output, the job-finding rate falls and income risk increases. This lowers demand, which is reflected in a lower natural rate of interest.

The right panel of Figure 7 compares the path of output implied by the natural rate paths shown in the left panel. The initial fall in employment in the complete markets model is 2.1 pct. points, compared to 5 pct. points in the incomplete markets model. Thus about 59% of the initial fall in employment in the full model is due to endogenous employment risk, while the rest is due to the initial exogenous shock. Equivalently, we may say that the fall in employment following from an exogenous reduction in demand is about 2.4 times greater than the exogenous reduction in demand itself, which is a sort of multiplier.

We can use the aggregate Euler equation (10) to decompose the amplification term into deleveraging and precautionary components. In section 4.2 we calculated this decomposition for the full model. To decompose the amplification term, we repeat this calculation for the natural rate in the flexible interest rate case, and then take the difference. This calculation reveals that about 53% of the amplification derives from deleveraging, with much of this concentrated in the first quarter. The remainder is due to increased precautionary saving.

Although the analysis in this section was restricted to exploring the dynamics of recovery from an initially wide asset distribution, these results suggest that higher moments of the asset distribution are important variables for central banks to monitor. Since households do not consider the effects of their borrowing decisions on aggregate employment in future liquidity trap states, the strong amplification mechanism identified in this section suggests that macroprudential policies aimed at reducing excessive debt accumulation would be welfare-increasing.[39] Since we can interpret the initial deleveraging as an event precipitated by a permanent credit shock (see the next section), these results also suggest that greater initial dispersion in asset holdings increases the sensitivity of the economy to financial shocks.[40]

4.4 Extensions to the Baseline Model

I consider two extensions to the baseline model, which are included in the appendix: microfounding the high-debt initial state in the deleveraging scenario, and the inclusion

[39] For an analysis of macroprudential policies in a model with a demand externality from household debt, see Korinek and Simsek (2014). For a general analysis of macroprudential policies in the presence of nominal rigidities, see Farhi and Werning (2013).

[40] This is similar to the finding of Heathcote and Perri (2014) that low levels of household wealth can increase macroeconomic volatility. In their model, this occurs because low wealth allows a low-employment equilibrium through a mechanism similar to the amplification process in this paper.

of unemployment insurance.

Microfounding the high-debt initial state. The baseline case considered an exogenous deleveraging scenario: the economy simply entered time $t = 0$ with a high debt asset distribution, depicted in figure 3, and then converged to the long-run equilibrium. One interpretation of this experiment is that the economy experienced an unanticipated permanent shock to credit conditions. Appendix A.1 formalizes this notion by explicitly modeling an unanticipated permanent decrease in the credit penalty D. The initial value of D is chosen to generate a steady state asset distribution with the same level of total debt as in the initial asset distribution in the baseline deleveraging experiment considered above. D then falls to the normal level, which tightens borrowing constraints and produces a deleveraging episodes. The resulting paths of all variables are essentially identical to the dynamics in the baseline case.[41] This correspondence suggests that the relevant details of the asset distribution for the deleveraging episode are well captured by the total debt, and other features of the asset distribution are of lesser importance.

Effect of including unemployment insurance. Since the amplification channel explored in this section operates through unemployment risk, we might expect unemployment insurance to mitigate amplification by reducing post-transfer income risk facing households. Appendix A.2 introduces unemployment insurance (financed by a tax on wage income) to the model and compares the dynamics of deleveraging to the baseline case without unemployment insurance. As expected, the presence of unemployment insurannce reduces the initial fall in employment due to the deleveraging episode. However, the presence of unemployment insurance also slows the pace of deleveraging, since the implied reduction in consumption volatility reduces the urgency of deleveraging for highly indebted households.

5 Deleveraging with Concurrent Credit Shock

The previous section considered a high-debt initial state that required a period of deleveraging. However, it is reasonable to think that the financial crisis that occurred in the Fall of 2008 represented a qualitatively different phenomenon than simply high levels of household debt. Arguably this was an exogenous credit shock, and lingering effects from the

[41] The only difference is that the initial fall in employment is about 0.75 pct. points greater than in the baseline case; but this lasts only a brief time (less than a quarter), and the path of employment coincides nearly exactly to the baseline case thereafter.

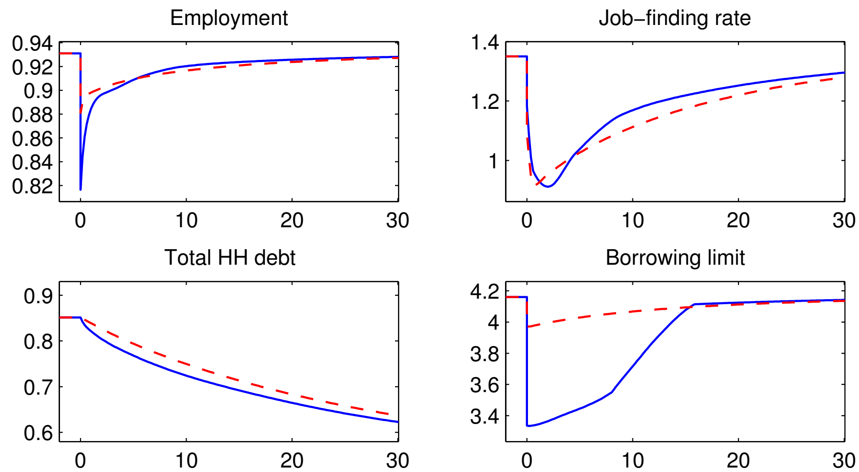

Figure 8: Deleveraging with credit shock (solid) vs. no shock (dashed).

financial crisis could also have contributed to the weakness of the recovery.

We can model this as an initial high level of household debt *together with* a temporary credit shock. In particular, suppose that initial household assets are as shown in Figure 3, and that entering period 0 the economy experiences an unanticipated temporary reduction in the default penalty D. This captures the idea of a credit shock in a stylized fashion, since a reduction in D will cause the borrowing constraint facing households to tighten. The resulting equilibrium reflects *both* the effects of a high level of initial debt that requires a period of deleveraging, and a temporary credit crunch facing households.

The resulting equilibrium is shown in Figure 8, with the baseline deleveraging experiment shown for comparison. The difference between the two cases is the path of the default penalty. In the credit shock case, the default penalty falls by 15% at $t = 0$, remains at this level for 8 quarters, and then linearly returns to the steady state value over another 8 quarters. While the details are not critical, the important point is that the exogenous shock is of significant size, but of shorter duration than the period of deleveraging in the baseline experiment.

The equilibrium path of employment is shown in the upper left panel of figure 8. The initial fall in employment is 11.4 pct. points, compared to a fall of 5 pct. points in the high debt case without a credit shock. The initial drop in employment is larger because there is a large mass of households close to the borrowing constraint. The fall in the default penalty causes the borrowing constraint to immediately tighten by about 19%, forcing these households to rapidly deleverage. This burst of deleveraging causes a sharp fall in demand, and therefore of employment.

This explanation is supported by the decomposition of the natural rate into deleveraging and precautionary terms, using the aggregate Euler equation (10). This decomposition reveals that 71% of the initial fall in employment is due to the deleveraging effect, compared to 45% in the baseline experiment. Thus suggests that most of the initial fall in demand is due to the behavior of constrained households. There is nevertheless a small increase in precautionary saving beyond the baseline case. This occurs because the tighter borrowing constraint reduces households' ability to smooth consumption, causing households throughout the asset distribution to increase precautionary saving.

Although the initial fall in demand is much greater than in the baseline case, the recovery is faster. The reason for this faster recovery is that deleveraging happens more quickly in the credit shock case, as can be seen from the bottom right panel of Figure 8, which shows the path of total debt. After 20 quarters, excess household debt has been reduced by 71% in the financial shock case, compared with 62% in the no shock case. Likewise, after 40 quarters 88% of deleveraging has been accomplished in the financial shock case, compared with 84% in the no shock case. This faster deleveraging is the consequence of the temporary credit shock, which forces highly indebted households to reduce their spending precipitously. This initially causes a much larger drop in demand, but when the credit shock begins to subside households have accomplished a greater amount of the necessary deleveraging.

The faster pace of deleveraging implies a much faster recovery in employment, as can be seen from the first panel in Figure 8. Although initial employment falls 2.3 times as far with the financial shock than without it, the employment path following the financial shock surpasses the no shock employment path after just 6 quarters, and remains higher thereafter. This leads to the surprising result that the cumulative loss of employment, and therefore output, is slightly *smaller* with the credit shock than in its absence. Cumulative employment loss in the credit shock case comes to 42.8 pct. points of employment, compared with 43.7 points in the no shock case. Thus the total decline in output is less in an economy with an additional adverse demand shock, if that shock speeds up the pace of deleveraging.[42]

This result is striking. It implies that the cumulative loss may be *less* following a *larger* demand shock. It is important to emphasize that this is not simply a larger demand shock by one measure; rather, the exogenous demand shock is *strictly* larger, at every point in time in every exogenous variable. The credit crisis, a substantial demand shock in its own right, was added to the same initial asset distribution that constituted the shock in

[42]Adjusting the magnitude and persistence of the credit shock may reverse this inequality, so that the cumulative employment loss in the credit shock case is greater than in the no shock case. However, the general result that the credit shock does not significantly increase cumulative employment loss is quite robust.

the baseline scenario, producing a significantly larger fall in initial employment, but lesser cumulative fall. This result suggests a view of financial crises similar to the liquidationist views of the business cycle, in which excessive accumulation of debt must be purged before growth can resume. To this end, a temporary credit crisis that accelerates this process may speed recovery, leading to less cumulative suffering, although greater employment losses in the short-run.[43]

6 Expectations and Recovery

Thus far we have assumed that the economy follows the path with the highest initial level of employment subject to the central bank's no-commitment policy rule. However, there are many other equilibrium paths that are consistent with the equilibrium conditions of the model. Which one the economy follows depends on the expectations of households, which may be influenced by the policy rule adopted by the central bank.

6.1 Forward Guidance

The analysis in sections 4 and 5 assumed that the central bank followed the no commitment policy rule after the economy exits the liquidity trap. However, if the central bank can credibly commit to allowing $n > n^*$ after the recovery, then an equilibrium path with higher employment is possible. If the central bank has a loss function that penalizes deviations from the target employment rate in both directions, then it would like to commit to allowing such a hiring boom. In this case, the central bank commits to allowing excessive hiring after the trap without raising interest rates.

This is a form of forward guidance, analogous to committing to keep interest rates low despite high inflation in a standard New Keynesian model. Here the cost of this policy is an inefficiently high level of output, which causes the marginal product of labor to fall below the wage. The expected future hiring boom raises demand in the liquidity trap for two reasons: first, the expectation of higher future consumption raises current demand due to consumption smoothing; second, the expectation of a high job-finding rate in the future reduces precautionary saving and raises demand.

[43] Rognlie, Shleifer, and Simsek (2014) discusses the history of the liquidationist perspective, and presents a model in which the stock to be liquidated is excessive aggregate capital accumulation, rather than excessive debt accumulated by some households.

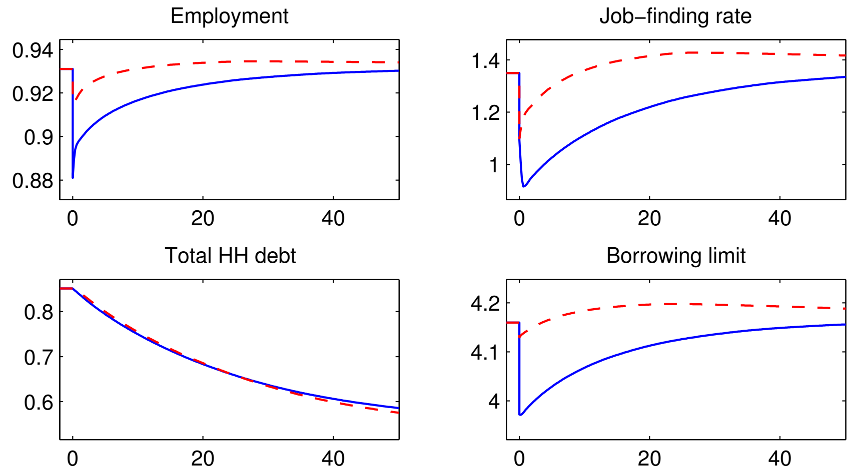

Figure 9: Hiring boom (dashed) vs. no commitment (solid).

Suppose the central bank targets a path of n to minimize the loss function

$$L = \int_0^\infty e^{-\rho t}[z(n(t))^{1-\alpha} - z(n^*)^{1-\alpha}]^2 dt \tag{29}$$

where n^* is the steady state level of employment. That is, the central bank seeks to minimize the discounted quadratic output gap.[44]

Figure 9 shows the dynamics of the economy in the baseline deleveraging experiment considered in section 4. The solid line shows the path of recovery when the central bank lacks commitment (identical to Figure 3), whereas the dashed line shows the equilibrium path under forward guidance, i.e. assuming the central bank chooses the path of n that minimizes (29). The forward guidance equilibrium allows a hiring boom in which the job-finding rate tops out at 1.43, 5.8% higher than in the no commitment case.

This higher job-finding rate reduces precautionary saving and loosens the borrowing constraint relative to the no-commitment equilibrium. The result is a substantially higher employment rate at every point in time. The initial fall in employment is 1.6 pct. points, just 31% of the fall in the no-commitment case. Employment recovers to its steady state level after about 10 quarters, and then overshoots the steady state, reaching a maximum level of 0.35 pct. points above steady state after about 28 quarters. Overall, the average level of employment over the entire course of recovery is about equal to steady state employment.

[44] A discounted quadratic loss function in the output gap and inflation is common in the New Keynesian literature because this is the linear approximation to the optimal policy rule around the efficient zero-inflation steady state. This rule is not optimal in the present model because of the presence of incomplete markets, but offers a simple benchmark to illustrate the power of forward guidance in this setting.

Thus forward guidance is capable of smoothing employment substantially relative to the no-commitment case. A simple measure of this is to compute the *absolute* cumulative employment deviation from trend. In the no-commitment baseline, total deviation is 43.7 pct. points - quarters of employment, compared to 24.4 under forward guidance. Moreover, the job-finding rate does not need to rise above the steady state by very much to achieve this result. This is the power of the amplification mechanism operating in a positive direction — just as a reduced job-finding rate produced a much lower path of employment than implied by excess debt alone, now the expectation of an elevated job-finding rate in the future increases demand today.

These results are in line with recent work showing powerful effects from forward guidance in New Keynesian models.[45] However, McKay, Nakamura, and Steinsson (2015) have shown that the presence of incomplete markets may *reduce* the power of forward guidance, since constrained households do not respond to news about future consumption. In my model, the inclusion of endogenous unemployment risk introduces another forward-looking term to the aggreagte Euler equation corresponding to a time-varying precautionary motive, as discussed in section 2.4. Thus forward guidance may be *more* effective at raising current consumption than in a complete markets model, since the expectation of a future boom reduces expected income volatility, causing unconstrained households to increase consumption.

These two effects correspond to the deleveraging and precautionary effects highlighted by the Euler equation decomposition in equation (10). In my baseline calibration, the precautionary effect dominates, and so forward guidance is more powerful than in an equivalent complete markets model. This result follows from two features of the model: first, relatively few households are actively constrained at any point in time, as seen in Figure 3; and second, income risk facing households is (endogenously) strongly procyclical. While determining which force predominates in the U.S. economy is ultimately an empirical question that this paper cannot clearly answer, there is some evidence that U.S. households face substantial procyclical income risk, which produce large swings in precautionary saving.[46]

6.2 Pessimistic Expectations About Recovery

The discussion above assumes that the economy always follows the highest path of employment consistent with the central bank's policy rule. However, there is no reason that

[45] See Eggertsson and Woodford (2003) and Carlstrom, Fuerst, and Paustian (2012).
[46] See Carroll, Slacalek, and Sommer (2012) and Parker and Preston (2005).

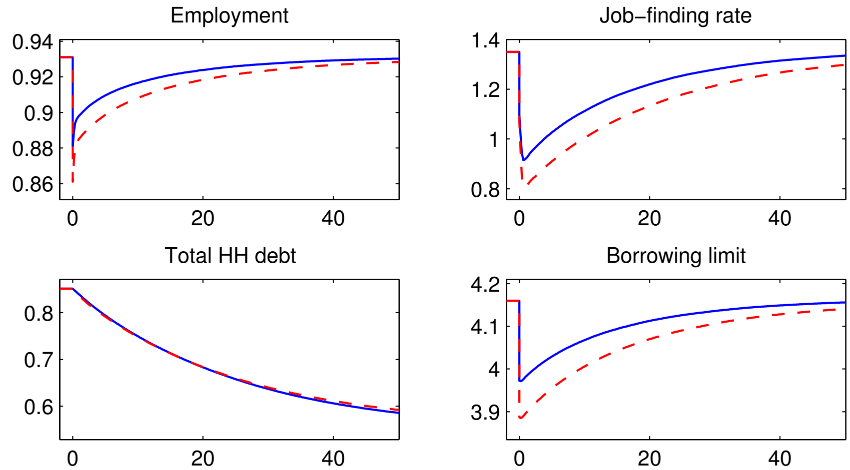

Figure 10: Pessimistic expectations (dashed) vs. baseline (solid).

this must be true. While the central bank can rule out equilibria with excessive hiring by raising interest rates, it has no analogous means to rule out low-employment equilibria because it is constrained by the lower bound on the interest rate. In the absence of commitment there is a continuum of equilibria, corresponding to different levels of initial employment $n(0)$.

Figure 10 shows a sample pessimistic equilibrium starting from the high-debt initial state analyzed in section 4, with the most optimistic no-commitment path shown for comparison. This particular equilibrium produces an initial fall in employment of 7 pct. points, 2 points greater than the highest no-commitment path. The paths slowly converge, with the pessimistic path of employment remaining more than 1 pct. point below the optimistic path for about 7 quarters, and coming within 0.5 pct. points after about 25 quarters.

Simultaneous to the slow recovery in employment is a period of slow hiring, which lowers demand and tightens the borrowing constraint. The job-finding rate remains well below the optimistic case throughout the period shown. The larger initial fall in the job-finding rate is by construction — this is exactly what it means in this model for households to be pessimistic about the rate of recovery, and this is the proximate cause of the larger initial fall in employment. Households' pessimistic expectations about the job-finding rate lower spending all along the path of recovery, both by increasing precautionary saving and by tightening the borrowing constraint. Thus households' pessimistic expectations are self-fulfilling, producing a larger initial fall in employment, and a slower recovery.

The bottom left panel of Figure 10 shows the path of total debt, and highlights a fur-

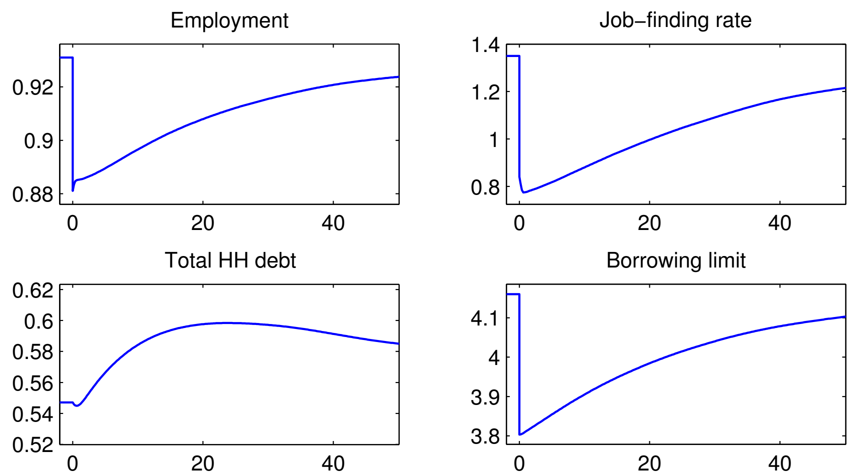

Figure 11: Pure expectational shock.

ther effect on the dynamics of the asset distribution. The lower rate of job-finding raises precautionary savings, while the lower employment rate increases the number of borrowers in the economy. This increases the dispersion of asset holdings relative to the baseline case, raising the total debt held by net borrowers panel and slowing the process of deleveraging. This greater dispersion of asset holdings weakens demand, further lowering the path of employment, although in the given case the effect is small relative to the total degree of deleveraging.

6.3 Pure Expectational Shock

Given the significant effects of pessimistic expectations on recovery following a shock (in this case, a high initial level of debt), it is natural to wonder what effect a pure expectational shock can produce. Suppose that the economy is at the steady state for $t < 0$, and at time 0 households come to expect a period of slow hiring. This will lead them to reduce their spending, decreasing demand and causing $n(0)$ to fall. The central bank cannot respond by lowering interest rates, and so it is powerless to prevent this fall in employment.

Under our maintained assumption that the economy converges to the steady state in the long-run, there is a unique path corresponding to each level of $n(0) < n^*$, with lower $n(0)$ corresponding to a larger shock to expectations. Figure 11 shows the path corresponding to a 5 pct. point drop in initial employment relative to the steady state. The result is a lengthy period of slow hiring. However, it is important to realize that the causality runs from the persistence to the magnitude of the initial drop in employment, rather than the reverse. In order to generate a 5 pct. point fall in current demand at time

$t = 0$, households must expect a very lengthy period of low job-finding. This must imply a similarly lengthy period with depressed employment, and a slow recovery.

Of particular interest are the dynamics of the asset distribution. As discussed above, a low job-finding rate increases asset dispersion because it stimulates greater saving from employed households while also increasing the total number of households that are borrowers (i.e. unemployed). This effect is sufficiently great that it offsets the narrowing of the asset distribution due to the tightening of the borrowing constraint, and significantly raises total debt held by borrowers over the course of the recovery, as shown in the bottom left panel of Figure 11. This greater asset dispersion plays a role in lowering demand. Intuitively, high unemployment worsens household balance sheets, which lowers demand and perpetuates high unemployment. Since the asset distribution is a relatively slow-moving variable (excepting initial deleveraging), this contributes to the slowness of recovery.

7 Conclusion

Unemployment risk significantly affects the dynamics of an economy in a liquidity trap. Unemployment risk both amplifies and increases the persistence of demand shocks, and allows multiple equilibria, corresponding to self-fulfilling optimism and pessimism about the pace of recovery. This effect derives from the feedback from slow hiring to weak demand, through precautionary savings and endogenous credit channels. Amplification is greater for more persistent shocks, such as deleveraging episodes. Additional demand shocks that facilitate faster deleveraging may reduce cumulative employment loss, at the cost of greater initial falls in employment. Persistence arises from the dynamics of the asset distribution, since a period of high unemployment raises the burden of debt, reducing demand during the recovery.

One important conclusion of this paper is that the distribution of debt is a critical variable in determining the dynamics of the economy. Moreover, it highlights the interaction between debt dynamics and precautionary effects. My results also suggest that a narrow focus on households that are actively constrained obscures the cumulative behavior of households throughout the asset distribution. Even households well away from the constraint will react to a tightening of credit access, because they desire a greater cushion of liquid assets, and any wealth shock or deterioration of labor market conditions will have greater effects on households with greater debt. These results suggest that central banks should monitor the distribution of debt, and take steps to reduce high household leverage. These results are particularly relevant given recent trends toward increased credit access and greater wealth inequality in the developed world, which may increase macroeconomic risk by increasing the fraction of households with low levels of wealth whose consumption is highly cyclically sensitive.

This paper also highlights the significant role of expectations in determining the path of recovery. These results suggest that if the central bank can credibly manage expectations about future policy, i.e. engage in forward guidance, it can significantly mitigate negative shocks. More ominously, these results also suggest the presence of self-fulfilling negative expectations about the future course of the economy. Given the zero lower bound on interest rates, the central bank may find itself powerless to prevent such outcomes. In light of slowing growth and demographic transitions around the world that depress the natural rate of interest, these possibilities are highly relevant to policymakers today.

I leave many questions to future research. This paper does not explicitly model durable goods and housing, although these constitute a large fraction of household credit use. Particularly given the centrality of mortgage debt and house prices in the 2007 – 2009 recession, explicitly considering such forms of debt could be a fruitful avenue of further research. In addition, this paper assumes that prices are fixed, which eliminates the role of inflation in equilibrium dynamics. Allowing for partially flexible prices could generate interesting interactions between inflation, unemployment risk, and the asset distribution.

Overall, my results suggest that a precautionary saving channel in response to high unemployment may have increased the magnitude of the 2007 – 2009 recession and the subsequent slow recovery. This analysis is complementary to explanations for the recession and slow recovery that highlight the role of household debt and deleveraging. It provides a mechanism through which the reduction in demand from a deleveraging episode is amplified. Further, this channel also helps explain the slow recovery, as high unemployment increased the burden of debt and slowed the process of deleveraging.

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A Online Appendix: Omitted Extensions

A.1 Permanent Credit Shock

The baseline case considered an exogenous deleveraging scenario: the economy simply entered time $t = 0$ with a high debt asset distribution, and then converged to the long-run equilibrium. One interpretation of this experiment is that the economy experienced an unanticipated permanent shock to credit conditions. This section formalizes this notion by explicitly modeling a permanent decrease in the credit penalty parameter D. The initial value of D is chosen to produce the same level of steady state debt as in the initial asset distribution in the baseline deleveraging experiment in section 4. This produces nearly identical dynamics to the baseline case, suggesting that the relevant details of the asset distribution are well captured by the total debt.

Since we are computing a steady state of the model with a higher D, steady state aggregate demand will be higher. In order to keep steady state n at its optimal level, the steady state interest rate will be higher. The new parameters that produce a steady state equilibrium with $n = n^*$, and total debt equal to the initial debt level in section 4 are shown in table 2.

D	$\underline{a}$	r
4.78	−6.24	0.35%

Table 2: Variables in steady state before permanent credit shock.

The resulting steady state asset distribution is shown in figure 12, with the initial high debt asset distribution used in section 4 shown for comparison. Total debt is the same in each distribution by construction, but the high D distribution has a wider support at both the high and low ends, and has a more symmetric shape, resembling the normal distribution, compared to the somewhat skewed hump-shaped distribution in the baseline case.

Transition Following a Permanent Credit Shock. Now suppose that at time $t = 0$, the default penalty D falls to the level in the baseline calibration. This represents a reduction in D of almost 31%, which implies a steady state constraint that is about 33% tighter than before. This produces a period of deleveraging, during which demand and therefore employment is depressed.

The paths of equilibrium variables are shown in figure 13, with the analogous paths from the baseline deleveraging experiment shown for comparison. Perhaps surprisingly,

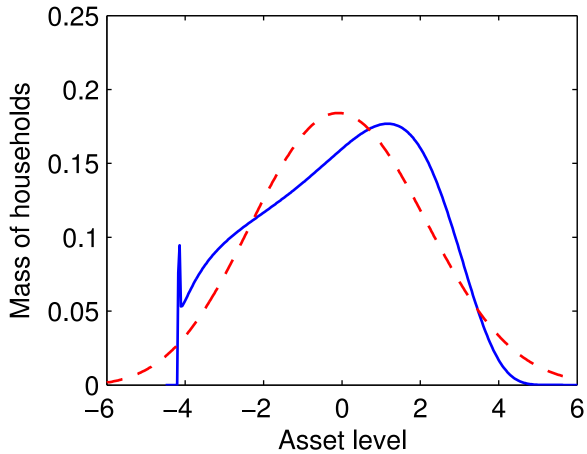

Figure 12: Baseline high debt (solid) vs. before permanent shock (dashed).

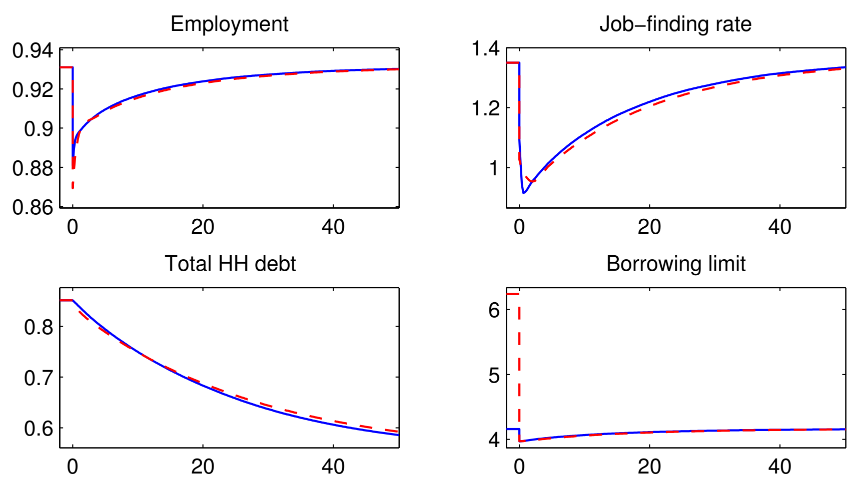

Figure 13: Baseline (solid) versus permanent credit shock (dashed).

the paths of all equilibrium variables are nearly identical. The primary difference is that the initial fall of employment is a bit larger — 6.2% instead of 5%. This greater initial fall in employment is due to the larger mass of households below the (new) borrowing constraint, as seen in the lower tail of the high-D steady state asset distribution in Figure 12. This increases the initial burst of deleveraging following the shock, causing demand (and therefore employment) to fall farther.

After this larger initial fall in employment, the subsequent paths of all equilibrium variables are essentially identical to the baseline experiment. After the somewhat greater initial burst of deleveraging, which lasts about one quarter, employment is equivalent. The initial levels of total household debt are the same by construction, and the subsequent evolution is likewise coincident.

The striking equivalence of the two cases, despite somewhat different asset distributions, suggests that the first two moments of the asset distribution are really what are most important in determining the effect on demand and the path of deleveraging. It further suggests that total debt is a reasonable proxy for the excess debt contained in the asset distribution, and that minor variations in the support, skewness, and other features of the asset distribution are unimportant.

A.2 Unemployment Insurance

Most advanced economies offer some form of unemployment insurance (UI) to workers. When workers lose their jobs, they receive payments from the government, which partially offset their lost wage income. One benefit of unemployment insurance is that it may dampen aggregate fluctuations by stabilizing aggregate demand, i.e. UI is a type of *automatic stabilizer*.[47] Since the amplification channel explored in this paper operates through endogenous unemployment risk, we might expect unemployment insurance to mitigate amplification by reducing post-transfer income risk facing households. This section investigates whether this is the case.

Unemployment insurance can be easily incorporated into the model framework. Suppose that unemployment insurance takes the form of a fixed benefit b paid to all unemployed workers, which is paid for with a lump-sum tax $\tau(t)$ paid by employed workers.

[47] For a discussion of automatic stabilizers in the U.S., and the channels through which they may have macroeconomic effects, see McKay and Reis (2013).

This is equivalent to adjusting the wage $w(t)$ and the nonlabor income $e(t)$ so that:

$$e^*(t) = e(t) + b \tag{30}$$

$$w^*(t) = w(t) - b - \tau(t) \tag{31}$$

where $\tau(t) = \left(\frac{1-n(t)}{n(t)}\right) b$. This nests the baseline specification with $b = 0$.

What happens if $b > 0$? This will increase nonlabor income, while simultaneously decreasing the wage. This will reduce the income differential between employed and unemployed households, which will tend to reduce c_e/c_u. Since the magnitude of the precautionary motive is decreasing in this ratio, as discussed in section 2.3, this will raise aggregate household spending for given parameters.

It is important to note that the unemployment insurance program described above is by construction revenue neutral; any increase in unemployment will prompt an increase in the payroll tax rate. By contrast, automatic stabilizers in many countries effectively act as automatic debt-financed fiscal policy. By focusing on a revenue-netural UI program, I isolate the precautionary and distributive channels of UI, while neglecting the fiscal stimulus component, which would require a quite different analysis.

Calibration. We would like to see what difference unemployment insurance makes in the process of deleveraging. Although we are chiefly interested in the transitional dynamics, unemployment insurance also affects the steady state. To facilitate a clean comparison, I calibrate new values of the parameters $\{\rho, D\}$, so that the steady state employment rate, borrowing constraint, and interest rate are the same as in the baseline case.

Table 3 shows the calibration corresponding to an unemployment benefit of size $b = 0.1$.[48] In steady state, this benefit raises the unemployed/employed income ratio from about 0.34 to about 0.41. Since the unemployment benefit reduces income volatility, it will reduce precautionary saving. Then households must become more patient to sustain the same aggregate saving rate in steady state, meaning that ρ must fall. Equivalently, we can say that introducing unemployment insurance raises the natural rate of interest, and to lower it again ρ must fall.

b	ρ	D
0.1	0.36%	3.20

Table 3: UI Calibration

[48]This corresponds to total expenditures on unemployment benefits equal to 0.48% of GDP. For comparison, according to McKay and Reis (2013) unemployment benefit transfers in the U.S. averaged about 0.33% of GDP over the postwar period.

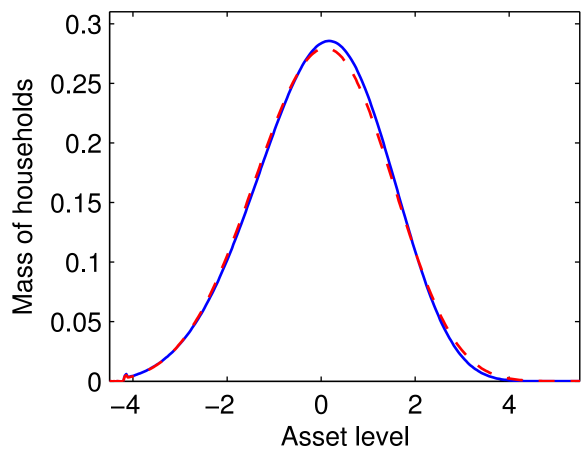

Figure 14: Baseline asset distribution (solid) vs. with UI (dashed).

Figure 14 compares the steady state asset distribution with and without unemployment insurance. The asset distribution is essentially identical in each case, suggesting that the change in parameters did not have some unanticipated effect on higher moments of the asset distribution.

Deleveraging with Unemployment Insurance. Suppose that the economy with unemployment insurance finds itself in the high-debt initial state used in the baseline deleveraging experiment in section 4. As in the baseline case, this produces a period of deleveraging, with depressed employment throughout due to reduced spending by highly indebted households.

The resulting paths of equilibrium variables are shown in Figure 15, with the baseline deleveraging experiment shown for comparison. Employment initially falls by 2.75 pct. points, less than the 5 pct. point drop in the baseline case. The smaller initial fall in employment is expected, and occurs because the presence of unemployment insurance dampens to amplification channel. Since unemployment insurance reduces the income differential between employed and unemployed households, it mitigates the effect on income variance of a given fall in the job-finding rate p. Thus for a given fall in p, the increase in the precautionary motive is smaller, implying a smaller fall in demand. This in turn implies a smaller fall in p than in the baseline case, producing a yet smaller further fall in demand, and so on. Thus the feedback from lower demand to lower p is dampened at every step of amplification, resulting in a smaller fall in p than in the baseline experiment, as can be seen in the top right panel of Figure 15.

Although the previous discussion highlighted the effect of unemployment insurance on precautionary saving, the effect also operates through the borrowing constraint. This

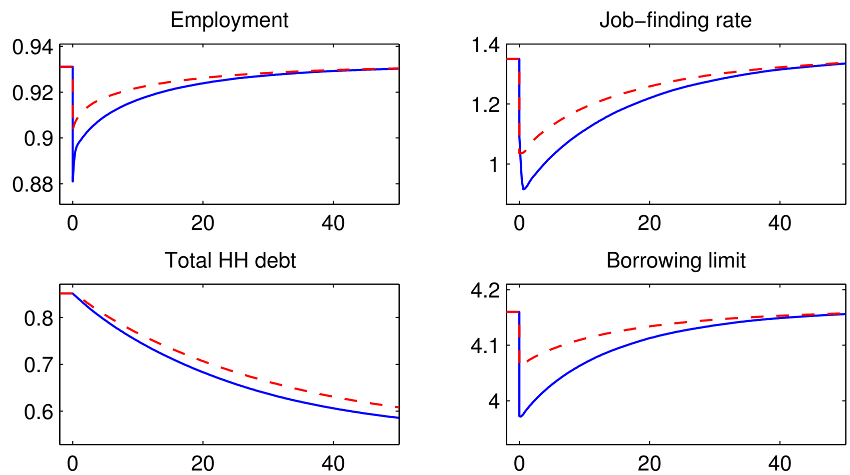

Figure 15: Equilibrium transition paths with UI (dashed) vs. without UI (solid).

can be seen in the lower right panel of Figure 15, which shows that the borrowing constraint tightens significantly less in the model with unemployment insurance than in the baseline model. This happens both because the job-finding rate does not fall by as much, and because a given fall in the job-finding rate tightens the borrowing constraint less. To see why the latter effect occurs, recall that a lower job-finding rate tightens the borrowing constraint by making unemployment spells longer. An unemployment spell is a period when the marginal value of wealth is relatively high, and therefore it is more costly to repay one's debts. Unemployment insurance reduces the differential in marginal value of wealth, and therefore makes the borrowing constraint less sensitive to swings in the job-finding rate. This result highlights a relatively neglected household credit channel through which unemployment insurance can stabilize demand. This is in line with the results of Hsu, Matsa, and Melzer (2014), who estimate that unemployment insurance expansions during the 2007 – 2009 recession prevented about 1.4 million foreclosures, and lead to increased credit supply to households.

Although the initial drop in employment is smaller in the model with unemployment insurance, the resulting recovery is slower, so that the paths of employment begin to converge well before the end of the recovery. This convergence occurs because the presence of unemployment insurance slows the process of deleveraging, as can be seen from comparing the paths of household debt in the bottom left panel of Figure 15. Deleveraging is slowed mainly by the smaller tightening of the borrowing constraint, and by the muting of the precautionary motive.

B Online Appendix: Omitted Derivations

B.1 Households

B.1.1 Household Euler equation

The costate equations of the Hamiltonian at $a > \underline{a}$ are

$$(\rho + s - r)\,\lambda = \lambda_a \dot{a}_e + \lambda_t + s\kappa$$
$$(\rho + p - r)\,\kappa = \kappa_a \dot{a}_u + \kappa_t + p\lambda$$

where $\lambda = V_a$ and $\kappa = U_a$ are the costate variables.

Let $\dot{\lambda} = \lambda_a \dot{a}_e + \lambda_t$ and $\dot{\kappa} = \kappa_a \dot{a}_u + \kappa_t$, so that $\dot{\lambda}(a,t)$ and $\dot{\kappa}(a,t)$ are the instantaneous rates of change of the costate variables of employed and unemployed households, respectively, when these households do not change employment status.[49] Using these terms, we can express the costate equations as

$$-\frac{\dot{\lambda}}{\lambda} = r - \rho + s\left(\frac{\kappa - \lambda}{\lambda}\right)$$
$$-\frac{\dot{\kappa}}{\kappa} = r - \rho + p\left(\frac{\lambda - \kappa}{\kappa}\right)$$

When $a > \underline{a}$, we can express the costate equations in terms of the consumption decision rules. First taking the expression in terms of $\dot{\lambda}$, and observing that $-\dot{\lambda}/\lambda = \gamma \dot{c}_e / c_e$, where $\dot{c}_e(a,t) = \frac{\partial}{\partial a} c_e(a,t) + \frac{\partial}{\partial t} c_e(a,t)$ is the instantaneous rate of change of consumption of an employed household that does not lose its job, we find that $\dot{c}_e$ satisfies

$$\frac{\dot{c}_e}{c_e} = \gamma^{-1}(r - \rho) + \gamma^{-1} s\left(\frac{c_u^{-\gamma}}{c_e^{-\gamma}} - 1\right) \tag{32}$$

where $\dot{c}_e(a,t) = \frac{\partial}{\partial a} c_e(a,t) + \frac{\partial}{\partial t} c_e(a,t)$ is the change in consumption by continuously employed households.

The term $\gamma^{-1} s\left(c_u^{-\gamma}/c_e^{-\gamma} - 1\right)$ captures the consumption response to unemployment risk. This term is positive because $c_u < c_e$, implying a faster rate of consumption growth corresponding to greater saving. This term does not perfectly capture the precautionary motive, however, because $\dot{c}_e$ only contains changes in consumption for households that remain employed, whereas we also should consider changes in consumption due to job

[49] This is an abuse of notation, since $\dot{c}$ refers to the rate of change of consumption by a particular household, rather than the evolution of the household decision rule defined at a particular point (a,t).

loss. In order to isolate the precautionary saving effect, we separate this expression into a precautionary term related to the *volatility* of future consumption, and a term corresponding to the *expected level* of future consumption. To do this, we simply add the expected change in consumption due to job loss to each side of (32) to obtain:

$$\underbrace{\frac{\dot{c}_e + s\left(c_u - c_e\right)}{c_e}}_{E[\dot{c}/c]} = \gamma^{-1}\left(r - \rho\right) + s\underbrace{\left[\gamma^{-1}\left(\frac{c_u^{-\gamma}}{c_e^{-\gamma}} - 1\right) - \left(1 - \frac{c_u}{c_e}\right)\right]}_{\text{precautionary motive}}$$

We may likewise express the Euler equation for unemployed households as:

$$\frac{\dot{c}_u + p\left(c_e - c_u\right)}{c_u} = \gamma^{-1}\left(r - \rho\right) + p\left[\gamma^{-1}\left(\frac{c_e^{-\gamma}}{c_u^{-\gamma}} - 1\right) - \left(1 - \frac{c_e}{c_u}\right)\right]$$

B.1.2 Approximation of precautionary saving term

We take a second-order Taylor expansion around $x = 1$. Then the precautionary motive term is approximately

$$T(x) \approx \frac{1}{2}\left(1 + \gamma\right) \times \left(1 - x\right)^2$$

Taking the problem of a currently employed household, let dc be the change in consumption of this household over a small time interval dt. Then for dt small, dc behaves like a binary random variable with probability distribution

$$dc = \begin{cases} \dot{c}_e \cdot dt & \text{with probability } 1 - sdt \\ c_u - c_e & \text{with probability } sdt \end{cases} \tag{33}$$

We used a similar concept above when we observed that $E\left[\dot{c}\right] = \left[\dot{c}_e + s\left(c_u - c_e\right)\right]$ in equation (6). Since $\dot{c} = dc/dt$, we can multiply by dt to obtain $E\left[dc\right] = \left[\dot{c}_e + s\left(c_u - c_e\right)\right] dt$, which gives the expectation of the random variable dc. We can likewise compute the variance, which, again neglecting higher order terms, is:

$$Var(dc) = s\left(c_u - c_e\right)^2 dt$$

or in terms of the rate of change in consumption, $s(c_u - c_e)^2 = Var(dc/dt) \cdot dt$.[50]

By analogy to the Euler equation, we are interested in the variance of the growth *rate* of

[50]Note that this implies that as $dt \to 0$, so that $dc/dt \to \dot{c}$, the variance of dc/dt approaches infinity. This reflects the possibility of discrete jumps in consumption following employment shocks.

consumption, dc/c. Thus we divide dc be c, which at time t is a known constant. Then we see that the Taylor expansion of the precautionary motive term for employed households is just:

$$sT\left(\frac{c_u}{c_e}\right) \approx \frac{1+\gamma}{2} \times Var\left(\frac{dc/dt}{c}\right) dt \tag{34}$$

B.1.3 Aggregate Euler equation

The household Euler equations (6) and (7) can be written as

$$E\left[\dot{c}\right] = \gamma^{-1}\left(r-\rho\right)c + qT\left(c_{-h}/c\right)\cdot c$$

where $T(x)$ is the precautionary motive term (8), q is the probability of an employment transition, and c_{-h} is consumption if the household switches employment status.

There is also a mass of constrained households, who are unemployed at asset level $\underline{a}$. If they remain unemployed, their consumption grows at $\dot{c} = -\ddot{\underline{a}}$, but if they become employed, which happens with probability pdt in time interval dt, they will increase their consumption by amount $c_e(\underline{a}) - c_u(\underline{a})$. Therefore their expected consumption growth is:

$$E\left[\dot{c}_{\underline{a}}\right] = p\left(c_e(\underline{a}) - c_u(\underline{a})\right) - \ddot{\underline{a}}$$

The expected rate of change of aggregate consumption $\dot{C}$ is just the weighted average of the expected changes of consumption of individual households. Moreover, because there is no aggregate risk in the economy, the actual growth of aggregate consumption equals its expectation. Therefore, letting χ be the share of households that are constrained, aggregate consumption growth satisfies

$$\frac{\dot{C}}{C} = \underbrace{(1-\chi)\,\gamma^{-1}\left(r-\rho\right)}_{\text{interest-rate substitution}} + \underbrace{T(\sigma_C^2)}_{\text{precautionary saving}} + \underbrace{\chi\left(\frac{p\Delta c(\underline{a}) - \ddot{\underline{a}}}{C}\right)}_{\text{constrained households}} \tag{35}$$

where $\Delta c(\underline{a}) = c_e(\underline{a}) - c_u(\underline{a})$ is the increase in consumption when a constrained household finds a job, and

$$T(\sigma_C^2) = \int\left[m_e\frac{c_e}{C}\cdot sT\left(\frac{c_u}{c_e}\right) + m_u\frac{c_u}{C}\cdot pT\left(\frac{c_e}{c_u}\right)\right] \tag{36}$$

is the consumption-weighted average of the precautionary saving terms of individual households.

We can derive a more intuitive expression for aggregate consumption growth using the second-order Taylor approximation of $T(x)$ given above. Let $\sigma_e^2(a,t)$ and $\sigma_u^2(a,t)$ be the consumption growth volatility facing a particular employed or unemployed household with assets a at time t. Then, as we showed above, $sT\left(c_u/c_e\right) \approx \left(\frac{1+\gamma}{2}\right)\sigma_e^2$ and $pT\left(c_e/c_u\right) \approx \left(\frac{1+\gamma}{2}\right)\sigma_u^2$. Then we can write the aggregate consumption Euler equation as:

$$\frac{\dot{C}}{C} \approx (1-\chi)\,\gamma^{-1}\,(r-\rho) + \left(\frac{1+\gamma}{2}\right)\sigma_C^2 + \chi\left(\frac{p\Delta c(\underline{a}) - \underline{\ddot{a}}}{C}\right) \tag{37}$$

where $\sigma_C^2 = \int_a \left[m_e \sigma_e^2 \frac{c_e}{C} + m_u \sigma_u^2 \frac{c_u}{C}\right]$ is the consumption-weighted average variance of the growth rate of consumption facing unconstrained households.

B.2 Final goods firms

Final goods are produced from intermediate goods using a Dixit-Stiglitz aggregation technology

$$Y = \left(\int_i y_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution between inputs. Consider the problem of a representative final good firm that purchases intermediate goods from intermediate good producers at prices p_i. The aggregator firm chooses inputs y_i to maximize profits

$$\Pi = P\left(\int_i y_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \int_i p_i y_i$$

which yields optimality condition

$$y_i = Y\left(p_i/P\right)^{-\epsilon}$$

Assuming zero profits, the aggregate price level is $P = \left(\int_i p_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$. The level of output is not pinned down by the price distribution, but is instead determined by the aggregate demand for final goods. Thus the final good producer takes aggregate demand as given, and purchases intermediate goods according to (15) to meet this demand. Since in equilibrium all intermediate good prices will be identical, the purchase of intermediate goods will satisfy $y_i = Y$.[51]

[51] All intermediate good prices will be identical because we will only consider the extremal cases of perfectly flexible or perfectly fixed prices. If prices were partially flexible, we would have price dispersion, with resulting aggregate productivity losses.

B.3 Computational Algorithm

I begin by solving for the steady state equilibrium. For given parameters, this involves finding the job-finding rate p and borrowing constraint $\underline{a}$ such that the stationary distribution of assets resulting from the household decision rules satisfies $A = 0$.

B.3.1 Household problem

I solve the household problem for given $(p, \underline{a})$ using value function iteration.[52] For given values of (V, U), we can define consumption using (4) and (5). The consumption rules define saving rules by (1). If the implied saving rule violates the boorrowing constraint, saving at $\underline{a}$ is set to zero.

Letting (V_n, U_n) be the value function at step n of value function, we can define the next step of the iteration (V_{n+1}, U_{n+1}) by

$$\rho V_{n+1} = u(c_e) + \frac{dV_{n+1}}{da} \cdot \dot{a}_e + \frac{V_n - V_{n+1}}{dt} + s\left(U_{n+1} - V_{n+1}\right)$$
$$\rho U_{n+1} = u(c_u) + \frac{dU_{n+1}}{da} \cdot \dot{a}_u + \frac{U_n - U_{n+1}}{dt} + p\left(V_{n+1} - U_{n+1}\right)$$

where dt is the step size. Note that this is equivalent to iterating backward through time, with dt equal to the time step. The system of equations above can be quickly solved for the next step by inverting a single matrix. This process continues until the process converges, which generally takes on the order of 20 iterations.[53]

B.3.2 Calculating the stationary distribution

Once we have calculated the decision rules of households, we can quickly solve for the implied stationary distribution of assets. From (23) and (24), the stationary asset distribution satisfies

$$pm_u - sm_e - \frac{d}{da}\left(m_e \dot{a}_e\right) = 0$$
$$sm_e - pm_u - \frac{d}{da}\left(m_u \dot{a}_u\right) = 0$$

To calculate the stationary distribution numerically we discretize the state space to a set of points a_i, and let $\Delta = a_{i+1} - a_i$ be the distance between adjacent points. Then we

[52] For a detailed description of this algorithm, see Achdou et al. (2013).

[53] Convergence of the algorithm requires an "upwind" approximation of the derivative of the value function dV/da, as described in Candler (2001).

approximate the derivative as

$$\frac{d}{da}(m\dot{a}) \approx \frac{1}{\Delta}\left[\max\left(m_{i-1}\dot{a}_{i-1}, 0\right) - \min\left(m_{i+1}\dot{a}_{i+1}, 0\right) - |m_i\dot{a}_i|\right]$$

This corresponds to taking the left difference when $\dot{a}_i > 0$, and the right difference when $\dot{a}_i < 0$. When $\dot{a}_i \approx 0$ so that $\dot{a}_{i-1} > 0$ and $\dot{a}_{i+1} < 0$, the expression corresponds to computing the first difference in the direction in which $m > 0$. This is important because $m_e\dot{a}_e$ may not be differentiable at a point where $\dot{a} = 0$, but the derivative in one direction will exist. Moreover, since we are in a two-state case, we know that $m = 0$ will hold in one direction when $\dot{a} = 0$. In particular, for employed households $m_{i+1} = 0$, and for unemployed households $m_{i-1} = 0$.

In addition to the expression above, we also have the requirement that $\sum_i m_i^e = \frac{p}{p+s} = N$ and $\sum_i m_i^u = \frac{s}{p+s} = 1 - N$. Note that in this case m_i is the mass at point i, rather than the pdf, which would equal m_i/Δ.

Now we write these equations in matrix form as $Tm = v$, where $m = \left(m_e^1, ..., m_e^n, m_u^1, ..., m_u^n\right)$, and v a vector with N in the nth position, and $1 - N$ in the 2nth position, and otherwise zero. (Here n is the number of grid points). T is a matrix with entries in terms of p, s, and $\dot{a}_i$, that encodes the rate of transitions above, but with the nth and 2nth row all 0s and 1s to encode the summations $\sum_i m_i^e = \frac{p}{p+s} = N$ and $\sum_i m_i^u = \frac{s}{p+s} = 1 - N$.

Then we simply invert the matrix T to find the stationary distribution $m = T^{-1}v$. This matrix is invertible because $\dot{a}(a)$ is strictly decreasing, and so equals zero at just one point.

B.3.3 Finding a transition path

Once we have the steady state, we can calculate the transition path following a shock as follows. First we guess a path of hiring probabilities $p(t)$ and initial employment $n(0)$. Given this path of hiring probabiltiies, we can iterate the household problem backward from the steady state using one step of the value function iteration algorithm used to solve for the steady state. At every step we must also calculate the borrowing constraint implied by the previously computed value function for unemployed households U.

Once we have the sequence of decision rules, we can iterate the initial asset distribution and employment forward to compute the path of asset demand. We then search for the path of hiring probabilities that makes asset demand zero at every point in time.

Because changing the hiring probability at any point in time has nonlinear effects on asset demand before and after this time, it is difficult to define a simple rule for updating the path of $p(t)$. Directly searching for the path of p that satisfied $A(t) = 0$ at every t is prohibitively computationally expensive. I instead search for values of p at several points, and interpolate the intermediate values, with more points immediately following the shock to capture the more complicated dynamics in this region. I confirmed that this method produces a very close approximation to $A = 0$, and varying the number of gridpoints at the margin does not alter the resulting dynamics.

B.4 Mean-preserving spread

The goal was to find a mean-preserving spread of the asset distribution that would preserve its approximate normality and leaving the support of the distribution unchanged. I did this as follows. Let the initial total asset distribution be $m(a)$, defined at gridpoints a_i. Then I defined the new spread asset distribution as

$$m_{\text{mps}}(a) = c_1 e^{c_2 (a - c_3)^2}$$

where the constants c_i are chosen to match three features of the resulting distribution: (1) target total debt held by households; (2) the mean of the assset distribution; (3) the total mass of households.